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Bayesian IGARCH Modeling of Jakarta Composite Index Volatility Using Hamiltonian Monte Carlo Algorithm

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Abstract
Time series models that model volatility in financial data, especially in stock market indices such as the Jakarta Composite Index (JCI), are Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. Following the ratification of the revised Armed Forces Law in March 2025, the JCI experienced increasing volatility, indicating persistent volatility. The problems in the JCI data require a time series model that can capture persistent volatility, namely the Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model. Parameter estimation for IGARCH models generally uses the Maximum Likelihood Estimation (MLE) method, which has limitations in handling parameter uncertainty. The Bayesian approach can address parameter uncertainty through the Markov Chain Monte Carlo (MCMC) methods. Among these, Hamiltonian Monte Carlo (HMC) is more efficient than Metropolis-Hastings and Gibbs Sampling, particularly in exploring complex posterior distributions. This study utilizes daily closing price data of the Jakarta Composite Index (JCI) as the main observation variable, observed from April 3, 2023, to April 9, 2025. This study aims to construct a volatility model for the Jakarta Composite Index (JCI) using a Bayesian IGARCH model with an HMC algorithm. This research only uses the IGARCH(1,1) model. The model has a strong ability to capture the JCI's volatility structure, and its point forecasts are stable. However, credible intervals reveal the uncertainty level, so the volatility of JCI may decrease or increase.

Keywords
Bayesian, GARCH, HMC, IGARCH, JCI

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1. INTRODUCTION

The main characteristics of financial time series is volatility, which reflects the unpredictable and variable nature of price movements over time. (Tsay, 2010). In practical terms, volatility refers to the magnitude of changes in asset prices (Floros, 2008). High volatility typically signals market turbulence, whereas low volatility often indicates periods of stability (Chaudhary et al., 2020). Analyzing and forecasting financial volatility is essential because it provides critical information for investors and policymakers. A commonly used method for modeling and forecasting financial market volatility is the Autoregressive Conditional Heteroskedasticity (ARCH) model which introduced by Engle (1982). To address the complexity of high-order ARCH models, Bollerslev (1986) developed the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which offers more flexible specifications for volatility dynamics (Enders, 2014). In financial econometrics, GARCH-type approaches have evolved into fundamental instruments for examining time series data that exhibit clustered volatility patterns (Kim et al., 2021). In practice, GARCH modeling is often performed on ARIMA model residual data after the ARCH effect has been identified (Xie et al., 2016).

Standard GARCH often assume that volatility shocks decay over time, making them unable to capture persistent volatility (Francq and Zakoian, 2019). To address the shortcoming of the GARCH model, Engle and Bollerslev (1986); Bollerslev (1986) developed the Integrated GARCH (IGARCH) model to address persistent volatility (Mills and Markellos, 2008). The IGARCH framework is highly suitable for financial time series that experience structural breaks or shifts in volatility regimes (Han and Park, 2014). Several studies have been conducted by Bentes (2021); González-Pla and Lovreta (2022); Montero et al. (2024); Maneejuk et al. (2025), who have successfully applied IGARCH to major global stock indices such as the S&P 500, FTSE, and DAX during periods of extreme volatility triggered by pandemics, natural disasters, and financial crises. These studies mostly focus on stock indices in developed countries, where financial systems are relatively mature and shocks to stock volatility caused by political policies, economic factors, and natural disasters can be quickly overcome. However, in developing countries, particulary in Indonesia, shocks from stock index volatility caused by political policies, economic policies, and other significant events tend to be highly reactive and have long-term effects. The Indonesian stock index referred to is the Jakarta Composite Index (JCI), an Indonesian stock index that measures the performance of all Indonesian stocks listed on the Indonesia Stock Exchange. According to Gunawan et al. (2022), the JCI is known for its volatility, which is easily shaken by internal shocks such as political and economic policies and other important events. Internal shocks to the volatility of the JCI occurred during the COVID-19 pandemic, and it is true that at the onset of COVID-19, the volatility of the JCI immediately increased (Alghifary et al., 2023). In addition to COVID-19 in 2020, there was an internal shock that caused the volatility of the JCI to increase in early 2025. In March 2025, the ratification of the revised Armed Forces Law sparked widespread public unrest and investor anxiety, leading to significant market turbulence (Pollard, 2025). The resulting sharp decline in the JCI and increased fluctuation in returns reflect persistent volatility, so need a model that can capture persistent volatility, namely IGARCH.

The IGARCH (1,1) parameter is generally estimated with the Maximum Likelihood Estimation (MLE) method. However, MLE only provides point estimates and does not take into account parameter uncertainty, which is very important in financial modeling. Financial data is highly volatile, which means there is an element of uncertainty, so a single parameter estimate is insufficient to describe the uncertainty in financial data. To address this shortcoming, can use Bayesian inference because it offers a more comprehensive framework by incorporating prior information and generating full posterior distributions. The posterior distribution obtained is several possible values of model parameter estimates, so that with the Bayesian method, several possible volatility values are obtained. Bayesian GARCH modeling can use MCMC methods such as Griddy-Gibbs Sampler and Metropolis–Hastings algorithms. The Griddy-Gibbs Sampler algorithm in GARCH models has been applied by in GARCH models has been applied by Ritter and Tanner (1992) and Xia et al. (2017) to estimate the Threshold GARCH model. The Metropolis–Hastings algorithm has been applied in Bayesian estimation of GARCH models by Dolmeta et al. (2023) and Chen et al. (2023) to estimate parameter asymmetric GARCH models. However, of the various MCMC techniques, the Hamiltonian Monte Carlo (HMC) algorithm in statistical analysis, applied by Neal (2011), is more efficient than Metropolis-Hastings and Gibbs Sampling, especially in exploring complex posterior distributions. HMC can reduce random walk behavior by incorporating gradient information that is usually seen in traditional MCMC methods (Liang et al., 2024). HMC algorithm has been used in GARCH models by Paixão and Ehlers (2017) on the GJR-GARCH model, (Burda and Bélisle, 2019) on the Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity (GJR-GARCH) and the GARCH Copula models, (Karmakar and Roy, 2020) in the tvARCH and tvGARCH model, (Goldman et al., 2023) on the DCC-GTARCH, DCC-GJR-GARCH, and DCC-GARCH model and the latest is Liang et al. (2024) on the Mixture Gaussian GARCH. Previous studies have shown that HMC is more efficient than other MCMC algorithms in exploring the parameter space, as demonstrated by faster convergence, lower autocorrelation between samples, and higher effective sample size than other MCMC algorithm (Betancourt, 2017; Hendriks et al., 2020; Liang et al., 2024; Neal, 2011; Perez-Roa et al., 2024; Yamada et al., 2022). Bayesian estimation based on HMC has been applied to many GARCH models, but has not yet been applied to IGARCH models. In addition, research on IGARCH-Bayesian modeling of stock index data in Indonesia, namely the JCI, where volatility is highly reactive to socio-political policies, is still very limited. Previous studies on JCI volatility modeling using GARCH models, namely GARCH, IGARCH, EGARCH, and GJR-GARCH models, generally used the MLE approach (Bahtiar, 2020; Fakhriyana et al., 2019; Nawatmi et al., 2023).

This study addresses this gap by modeling IGARCH estimated using the Bayesian HMC algorithm method to analyze volatility in the JCI, particularly during the turbulence following the ratification of Indonesia's revised Armed Forces Law in March 2025, and then forecast volatility of JCI after the turbolence. This research can provide insights for market participants in conditions of JCI uncertainty. However, this research has limitations in that it only uses the GARCH(1,1) model. The GARCH(1,1) model was used because of its simple parameterization, which minimizes the number of estimated parameters and prevents significant loss of degrees of freedom.

2. EXPERIMENTAL SECTION

2.1 Materials

This study uses daily closing price data from the Jakarta Composite Index (JCI) from April 3, 2023, to April 9, 2025. The data were obtained from https://finance.yahoo.com. For the modelling process, daily returns were calculated from price data using the continuously compounded return formula in Equation (1) (Brooks, 2014).

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1}) \quad (1)$$

In Equation (1), r_t is the JCI return, P_t is the JCI index at time-t, and P_{t-1} is the JCI index at time-$(t-1)$.

2.2 Methods

This study estimates the parameters of the Bayesian IGARCH model using Hamiltonian Monte Carlo (HMC). However, before modeling IGARCH, conduct ARIMA modeling on JCI returns data, then test the ARCH effect for ARIMA model residuals. In financial econometrics, returns are preferred over raw prices since asset prices are usually non-stationary, while returns better capture relative market movements and volatility (Li, 2023). Prior to applying the IGARCH (1,1) model, an ARIMA process is fitted to the return series to capture and remove any autocorrelation in the mean equation (Tsay, 2010).

2.2.1 ARIMA (Autoregressive Integrated Moving Average)

ARIMA integrates AR and MA terms with differencing to make a non-stationary time series stationary (Wei, 2006). The ARMA(p,q) models is presented in Equation (2), and the ARIMA (p, d, q) model is presented in Equation (3) (Cryer and Chan, 2008).

$$\begin{gathered}\phi(B)r_t = \theta(B)e_t \\ (1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)r_t = (1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q)e_t \\ r_t-\phi_1 r_{t-1}-\cdots-\phi_p r_{t-p} = e_t-\theta_1 e_{t-1}-\cdots-\theta_q e_{t-q}\end{gathered} \quad (2)$$

$$\begin{gathered}\phi(B)(1-B)^d r_t = \theta(B)e_t \\ (1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)(1-B)^d r_t = (1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q)e_t\end{gathered} \quad (3)$$

In Equation (2) and Equation (3), r_t is the return at time-t, B is the backshift operator, defined as $Br_t = r_{t-1}$, $B^2 r_t = r_{t-2}$, and $B^p r_t = r_{t-p}$. The coefficients ϕ_i for $i = 1, 2, \ldots, p$ are the autoregressive (AR) parameters, θ_j for $j = 1, 2, \ldots, q$ are the moving average (MA) parameters, p is the AR order, q is the MA order, and e_t is the residual at time-t, assumed to be white noise with zero mean and constant variance, namely σ_e^2. The parameter d in Equation (3) is the order of differencing used to make the time series data stationary.

Estimation of ARIMA model parameters using the Maximum Likelihood Estimation (MLE) method (Cryer and Chan, 2008). Before fitting the ARIMA model, stationarity of both variance and mean must be tested. Variance stationarity is assessed using the Box-Cox lambda value (Kusdarwati et al., 2022), while mean stationarity is evaluated through the Augmented Dickey–Fuller (ADF) test (Cryer and Chan, 2008). After parameter estimation, residual diagnostics are conducted to examine autocorrelation using the Ljung–Box statistic test and to assess normality through the Jarque–Bera test (Hanke and Wichern, 2014). However, the normality assumption is not strictly necessary in financial time series modeling (Hyndman and Athanasopoulos, 2018). The best model selection is done by looking at AIC value (Cryer and Chan, 2008).

2.2.2 Model ARCH, GARCH, and IGARCH

According to Cryer and Chan (2008), the ARCH model is used to analyze time series data that contains volatility. ARCH model is defined in Equation (4) (Box et al., 2015).

$$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \cdots + \alpha_s e_{t-s}^2 \quad (4)$$

In Equation (4), σ_t^2 is the variance at time-t, ω is a constant component, α_1 is the ARCH parameter, and e_{t-s}^2 is the squared residual from the ARIMA model at time-$t-s$. Note that ω and α_s must satisfy ≥ 0.

However, a limitation of ARCH is the requirement for a high lag order q. To address this, the GARCH (s,r) model, introduced by Bollerslev (1986), incorporates lagged conditional variances. The GARCH (s,r) model is shown in Equation (5) (Box et al., 2015).

$$\sigma_t^2 = \omega + \sum_{i=1}^{s} \alpha_i e_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 \quad (5)$$

In Equation (5), β_j for $j = 1, 2, \ldots, r$ is a parameter of GARCH, α_i for $i = 1, 2, \ldots, s$ is a parameter of ARCH, and ω is a constant parameter.

Before applying the model, ARCH effects are detected using the Lagrange Multiplier (LM) test (Tsay, 2010). However, in this study only use GARCH (1,1) model which defined in Equation (6) (Tsay, 2010).

$$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (6)$$

Estimation of GARCH(1,1) model parameters using the MLE method (Cryer and Chan, 2008). If the results show that the sum of α_1 and β_1 is equal to one, then there is an indication of persistent volatility. Therefore, can use the IGARCH (1,1) which defined in Equation (7) (Tsay, 2010).

$$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (7)$$

2.2.3 Bayesian Concept in the IGARCH Model

According to Ardia (2008), Bayesian GARCH modeling assumes a parameter vector $\boldsymbol{\delta} = [\delta_1, \delta_2, \delta_3]$, where $\delta_1 = \omega$, $\delta_2 = \beta_1$, and $\delta_3 = \alpha_1$, with the constraint $\alpha_1 = (1 - \beta_1)$. as required by the IGARCH (1,1) structure. In the Bayesian approach, the posterior distribution is formed by updating prior beliefs using observed data through the likelihood function (Gelman et al., 2019). The posterior distribution is shown in Equation (8) (Gelman et al., 2019).

$$p(\boldsymbol{\delta} \mid \mathbf{e}) = \frac{p(\boldsymbol{\delta})\, L(\mathbf{e} \mid \boldsymbol{\delta})}{p(\mathbf{e})} \quad (8)$$

In Equation (8), $p(\delta)$ presents the prior distribution, p(e) presents the marginal likelihood, and L($e \mid \delta$) denotes the likelihood function derived from the residuals. The likelihood function, as described by Karmakar and Roy (2020), is presented in Equation (9).

$$L(\mathbf{e} \mid \boldsymbol{\delta}) \propto |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\mathbf{e}'\boldsymbol{\Sigma}^{-1}\mathbf{e}\right) \quad (9)$$

In Equation (9), $e = \begin{bmatrix} e_t & e_{t-1} \\ \vdots & \vdots \\ e_{t-n+1} & e_{t-n} \end{bmatrix}'$, and Σ in Equation (9) is defined in Equation (10) and its matrix elements are provided in Equation (11) (Ardia, 2008).

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_t^2 & \cdots & \sigma_{t,t-n} \\ \vdots & \ddots & \vdots \\ \sigma_{t-n,t} & \cdots & \sigma_{t-n}^2 \end{bmatrix} \tag{10}$$

$$\sigma_{t-i}^2 = \omega + (1-\beta_1)e_{t-i-1}^2 + \beta_1\sigma_{t-i-1}^2 \tag{11}$$

The off-diagonal elements $\sigma_{(t-i,t-j)}$ in Equation (10) represent conditional covariances between residuals at different lags. If the residuals are assumed to be uncorrelated, as is typical in IGARCH models, Σ in Equation (11) becomes a diagonal matrix with σ_{t-i}^2 on the main diagonal, so the likelihood function which is defined in Equation (9) is redefined in Equation (12) (Cryer and Chan, 2008).

$$L(\mathbf{e} \mid \boldsymbol{\delta}) \propto \left(\prod_{t=1}^{n} \frac{1}{\sqrt{\sigma_t^2}}\right) \exp\left(-\frac{1}{2}\sum_{t=1}^{n}\frac{e_t^2}{\sigma_t^2}\right) \tag{12}$$

The prior distribution for the parameter δ assumed that δ follows a independent gamma distribution. The prior distribution function for δ is defined in Equation (13) (Hsieh, 2023).

$$\begin{aligned} p(\boldsymbol{\delta}) &= p(\delta_1)\,p(\delta_2)\,p(\delta_3) \\ &= \begin{cases} \left(\dfrac{1}{\beta_{\delta_1}^{\alpha_{\delta_1}-1}\Gamma(\alpha_{\delta_1})}\delta_1^{\alpha_{\delta_1}-1}e^{-\delta_1/2}\right)\left(\dfrac{1}{\beta_{\delta_2}^{\alpha_{\delta_2}-1}\Gamma(\alpha_{\delta_2})}\delta_2^{\alpha_{\delta_2}-1}e^{-\delta_2/2}\right), & \boldsymbol{\delta} > 0, \\ 0, & \boldsymbol{\delta} \leq 0. \end{cases} \end{aligned} \tag{13}$$

The prior structure can also use a independent gamma distribution as defined in Equation (13). The use of a gamma prior in Bayesian estimation of GARCH or IGARCH models is theoretically justified, as these models represent conditional variances that evolve over time. Because variance is inherently non-negative, its prior distribution must be defined on the positive real domain. According to Wackerly (2014), the variance of a normally distributed variable follows a Chi-Square distribution, a special case of the Gamma distribution with shape $\alpha = v/2$ and scale parameter $\beta=2$ (Hsieh, 2023). The values of α and β the Gamma and Chi-Square distributions must also be positive (Wackerly, 2014). Positive values of α and β can ensure that the parameter values in the GARCH model are positive. However, the Chi-Square distribution has limited flexibility because its parameters are strictly tied to the degrees of freedom, which restricts the control over its mean and variance. Unlike the Chi-Square, the Gamma prior offers greater flexibility in defining (α) and (β) parameters independently allowing for more valid estimation results. The posterior distribution derived from this formulation is approximated using the HMC algorithm but also using the Metropolis Hastings (MH) algorithm to compare effectiveness from in exploring the posterior distribution. In each iteration using HMC and MH algorithm the prior parameters are fixed.

2.2.4 Hamiltonian Monte Carlo Algorithm (HMC)

Initially, the Hamiltonian Monte Carlo (HMC) algorithm was developed in molecular dynamics (Duane et al., 1987) and later introduced to statistics by Neal (2011). According to Betancourt (2017) and Girolami and Calderhead (2011), the HMC method enhances sampling efficiency by leveraging gradient information, which directs the sampler through the parameter space more effectively and mitigates random walk behavior. The algorithm represents the parameter space through Hamiltonian dynamics, pairing each parameter vector δ with an auxiliary momentum vector ϕ of the same dimension (Han and Park, 2014). The Hamiltonian function $\mathrm{H}(\delta,\phi)$ is defined in Equation (14) (Neal, 2011).

$$H(\mathbf{\Phi}, \boldsymbol{\delta}) = K(\mathbf{\Phi}) + U(\boldsymbol{\delta}) \tag{14}$$

In a statistical model, Φ in Equation (14) represents the momentum parameter vector, with each element corresponding to a parameter in the model. δ in Equation (14) denotes the vector of model parameters (e.g., ω, β_1, α_1). $K(\Phi)$ in Equation (14) is defined as the negative log of the momentum density and is usually assumed to follow a multivariate normal distribution. $U(\delta)$ in Equation (14) represents the negative logarithm of the posterior probability density (Hanada, 2022). The Hamiltonian function is redefined in Equation (15) (Neal, 2011).

$$\begin{aligned} H(\mathbf{\Phi}, \boldsymbol{\delta}) &= -\ln(p(\mathbf{\Phi})) - \ln(p(\boldsymbol{\delta})) \\ H(\mathbf{\Phi}, \boldsymbol{\delta}) &= -\ln\left(\exp\left(\tfrac{1}{2}\mathbf{\Phi}'\mathbf{\Sigma_\Phi}\mathbf{\Phi}\right)\right) - \ln(p(\mathbf{e} \mid \boldsymbol{\delta})\,p(\boldsymbol{\delta})) \end{aligned} \tag{15}$$

Equation (15) explains that the Hamiltonian is the integration of the negative logarithm of the posterior density with the negative logarithm of momentum, which forms the basis of the Hamiltonian Monte Carlo (HMC) sampling method. The Hamiltonian allows the algorithm to generate vector fields that are parallel to the posterior distribution region. The vector field is derived from the Hamiltonian equations, which are presented in Equation (16) (Betancourt, 2017).

$$\begin{aligned} -\frac{\partial H(\mathbf{\Phi}, \boldsymbol{\delta})}{\partial \delta_i} &= -\frac{\partial U(\boldsymbol{\delta})}{\partial \delta_i} = -\frac{\partial}{\partial \delta_i}\left(-\ln(p(\mathbf{y} \mid \boldsymbol{\delta})\,p(\boldsymbol{\delta}))\right) \\ -\frac{\partial H(\mathbf{\Phi}, \boldsymbol{\delta})}{\partial \Phi_i} &= -\frac{\partial K(\mathbf{\Phi})}{\partial \Phi_i} = -\frac{\partial}{\partial \Phi_i}\left(\tfrac{1}{2}\,\mathbf{\Phi}'\mathbf{\Sigma_\Phi}\mathbf{\Phi}\right) \end{aligned} \tag{16}$$

According to Betancourt (2017), $\frac{\partial H(\Phi,\delta)}{\partial \delta_i}$ in Equation (16) denotes the posterior distribution gradient, guiding updates of the momentum and target distributions in the leapfrog iteration. The iteration begins by initializing both the momentum vector and the target parameter vector, denoted as $\Phi_0 = \Phi$ and $\delta_0 = \delta$. Following this, a series of leapfrog iterations is conducted using the number of step adjustments. The first step involves updating the momentum parameters with a half-step adjustment, using the gradient of the target parameter vector as described in Equation (17).

$$\boldsymbol{\Phi}_{n+\frac{1}{2}} = \boldsymbol{\Phi}_n - \frac{T}{2L}\frac{\partial U(\boldsymbol{\delta}_n)}{\partial \delta_i} \quad (17)$$

Next, the parameter momentum obtained by Equation (17) is used to update the one-step target parameter defined in Equation (18) (Betancourt, 2017).

$$\boldsymbol{\delta}_{n+1} = \boldsymbol{\delta}_n + \frac{T}{L}\boldsymbol{\Phi}_{n+\frac{1}{2}} \quad (18)$$

Finally, the momentum is updated again using Equation (19) with the new gradient from of the target parameter vector which obtained by Equation (18).

$$\boldsymbol{\Phi}_{n+1} = \boldsymbol{\Phi}_n - \frac{T}{2L}\frac{\partial U(\boldsymbol{\delta}_{n+1})}{\partial \delta_i} \quad (19)$$

After completing these steps, the proposal (Φ_L, δ_L) is obtained and then either accepted or rejected by the probability Pr as defined in Equation (20) (Betancourt, 2017).

$$Pr = \min\left[1, \frac{\pi(\boldsymbol{\Phi}_L, \boldsymbol{\delta}_L)}{\pi(\boldsymbol{\Phi}, \boldsymbol{\delta})}\right] = \min\left[1, \exp\left(-H(\boldsymbol{\Phi}_L, \boldsymbol{\delta}_L) + H(\boldsymbol{\Phi}, \boldsymbol{\delta})\right)\right] \quad (20)$$

To ensure that the Hamiltonian Monte Carlo sampling process reached a stable posterior distribution, convergence diagnostics were performed (Betancourt, 2017). Convergence evaluation in MCMC algorithms, including HMC, involves evaluation of trace plots, ergodic mean plots, kernel density plots, ACF, Monte Carlo Error (MC Error), Gelman-Rubin Statistic, and Effective Sample Size (ESS) (Karmakar and Roy, 2020).

2.2.5 Hamiltonian Monte Carlo (HMC) Algorithm in the IGARCH Model

In the Hamiltonian function for the IGARCH model, δ represents the target parameter corresponding to the GARCH model parameters. Φ denotes the momentum vector, sharing the same dimensionality as the target parameter. Φ is multivariate normally distributed with mean vector $\mu = 0$ and covariance matrix $\Sigma = \Sigma_\Phi$ (Betancourt, 2017). The Hamiltonian function of the IGARCH model is specified in Equation (21). This formulation adheres to the general Hamiltonian Monte Carlo framework for GARCH models (Burda and Bélisle, 2019; Liang et al., 2024), but the key difference is that this study uses the IGARCH (1,1) model.

$$\begin{aligned} H(\boldsymbol{\Phi}, \boldsymbol{\delta}) &= K(\boldsymbol{\Phi}) + U(\boldsymbol{\delta}) = -\ln\left(\exp\left(\tfrac{1}{2}\boldsymbol{\Phi}'\boldsymbol{\Sigma_\Phi}\boldsymbol{\Phi}\right)\right) - \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right) \\ H(\boldsymbol{\Phi}, \boldsymbol{\delta}) &= \tfrac{1}{2}\boldsymbol{\Phi}'\boldsymbol{\Sigma_\Phi}\boldsymbol{\Phi} - \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right) \end{aligned} \quad (21)$$

In Equation (21), $\ln\left(p(\delta \mid e)\right)$ is the log target distribution function for the parameters of the IGARCH(1,1) model, and the log-likelihood for the IGARCH(1,1) model is defined in Equation (22).

$$\begin{aligned} \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right) &= \ln\left(p(\boldsymbol{\delta})\, L(\mathbf{e} \mid \boldsymbol{\delta})\right) \\ \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right) &= \ln\Big((\delta_1^{\alpha_{\delta_1}-1}\exp(-\delta_1/2))(\delta_2^{\alpha_{\delta_2}-1}\exp(-\delta_2/2)) \\ &\qquad |\boldsymbol{\Sigma}|^{-1/2}\exp\left[-\tfrac{1}{2}\mathbf{e}'\boldsymbol{\Sigma}^{-1}\mathbf{e}\right]\Big) \end{aligned} \quad (22)$$

Assumed that no correlation between residuals at different lags, so ln target distribution function for parameters IGARCH (1,1) in Equation (22) can be redefined in Equation (23)

$$\begin{aligned} \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right) &= \ln\left(p(\boldsymbol{\delta})\, L(\mathbf{e} \mid \boldsymbol{\delta})\right) \\ \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right) &= \ln\Big((\delta_1^{\alpha_{\delta_1}-1}\exp(-\delta_1/2))(\delta_2^{\alpha_{\delta_2}-1}\exp(-\delta_2/2)) \\ &\qquad \cdot\Big(\prod_{t=1}^{n}\frac{1}{\sqrt{\sigma_t^2}}\Big)\exp\Big[-\tfrac{1}{2}\sum_{t=1}^{n}\frac{e_t^2}{\sigma_t^2}\Big]\Big) \end{aligned} \quad (23)$$

In the IGARCH model, the gradient of the kinetic (momentum) component of the Hamiltonian function, denoted as $\frac{\partial H(\Phi,\delta)}{\partial \Phi_i}$, is given by $\Sigma_\Phi^{-1}\Phi$. The gradient of the target distribution component of the Hamiltonian function, denoted as $\frac{\partial H(\Phi,\delta)}{\partial \delta_i}$, is obtained through the formulation defined in Equation (24).

$$\begin{aligned} \frac{\partial H(\boldsymbol{\Phi}, \boldsymbol{\delta})}{\partial \delta_i} &= \frac{\partial \ln\left(p(\boldsymbol{\delta} \mid \mathbf{e})\right)}{\partial \delta_i} \\ \frac{\partial H(\boldsymbol{\Phi}, \boldsymbol{\delta})}{\partial \delta_i} &= \frac{\partial}{\partial \delta_i}\ln\Big((\delta_1^{\alpha_{\delta_1}-1}\exp(-\delta_1/2))(\delta_2^{\alpha_{\delta_2}-1}\exp(-\delta_2/2)) \\ &\qquad \cdot\Big(\prod_{t=1}^{n}\frac{1}{\sqrt{\sigma_t^2}}\Big)\exp\Big[-\tfrac{1}{2}\sum_{t=1}^{n}\frac{e_t^2}{\sigma_t^2}\Big]\Big) \end{aligned} \quad (24)$$

Equation (24) is utilized in the leapfrog iterations to generate updated values for both the target distribution and the momentum distribution which defined in Equation (17) and Equation (18). Equation (24) is the partial derivative of σ_t^2 in the parameters of the IGARCH. The partial derivative concerning one of the parameters, namely ω, is defined in Equation (25).

$$\frac{\partial U(\boldsymbol{\delta})}{\partial \omega} = \sum_{t=\max(p+1,\,q+1)}^{n}\frac{\partial \sigma_t^2}{\partial \omega} \quad (25)$$

The partial derivative in Equation (25) also applies to the other parameters in the IGARCH(1,1) model, namely α_1 and β_1, to obtain the gradient of the target distribution.

2.2.6 Forecasting

Volatility forecasting is performed using parameter estimates derived from the Bayesian HMC. Equation (26) specifies the variance forecast for time the variance forecast for time t+1 (Tsay, 2010).

$$\hat{\sigma}^2_{t+1} = \omega + (1 - \beta_1)e^2_t + \beta_1\hat{\sigma}^2_t \quad (26)$$

In Equation (26), $\hat{\sigma}^2_t$ is calculated by taking the average of the variance estimated using all IGARCH (1,1) model parameters obtained in each iteration. $\hat{\sigma}^2_t$ is calculated using Equation (27) (Liang et al., 2024).

$$\hat{\sigma}^2_t = \frac{1}{N}\sum_{m=1}^{n}\hat{\sigma}_t^2(m) \quad (27)$$

In Equation (27), N is the number of posterior samples. For each sample m, the variance $\sigma_t^2(m)$ is computed using Equation (28) (Liang et al., 2024).

$$\hat{\sigma}_t^2(m) = \hat{\omega}^{(m)} + \left(1 - \hat{\beta}_1^{(m)}\right)e^2_{t-1} + \hat{\beta}_1^{(m)}\sigma^2_{t-1} \quad (28)$$

In Equation (28), $\hat{\omega}^{(m)}$ and $\hat{\beta}_1^{(m)}$ are the values of the model parameters obtained from the m-th posterior draw. The averaging across all samples ensures that parameter uncertainty is fully incorporated into the forecast. Multi-step-ahead volatility forecasts are generated recursively. For forecast horizon $l > 1$, the predictive variance is updated using Equation (29).

$$\hat{\sigma}^2_{t+l} = \hat{\sigma}^2_{t+l-1} + (l-1)\,\omega, \quad \text{for } l \geq 2 \quad (29)$$

3. RESULTS AND DISCUSSION

The first thing to do when analyzing time series is to visualize the data to detect structural patterns and shifts. In this study, time series graphs were used to observe fluctuations in the Jakarta Composite Index (JCI) and its daily returns from April 2023 to April 2025 which obtained using Equation (1). Figure 1 displays both the closing price series and the return series over this period.

Figure 1(a) shows that the Jakarta Composite Index (JCI) is likely to move upward from April 2023 to December 2024. However, in 2025, the index began to decline sharply. This decline was likely triggered by several key developments in Indonesia's economy and political climate namely government policies related to budget efficiency. This policy had a negative impact on several state-owned companies and led to a decline in stock prices The second policy occurred in March 2025, namely the revision of the Indonesian Military Law, which triggered widespread public dissatisfaction and massive demonstrations in various major cities. These disruptions prompted the IDX to suspend trading, leading to a substantial drop in the JCI. Figure 1(b) shows that the JCI returns fluctuated significantly from April 2023 to February 2025, with the fluctuations becoming even more pronounced during March and April 2025. This time series pattern suggests the presence of persistent volatility. To validate this observation, it is necessary to fit an ARIMA model and test ARCH effects for residuals.

3.1 ARIMA Modelling

The variance stationarity test on the JCI return data produced a Box-Cox transformation parameter value of $\hat{\lambda} = 1.03503$, indicating that $\hat{\lambda}$ is close to one. Therefore, it can be concluded that the JCI returns are stationary in variance. The stationarity of the series mean was examined using the Augmented Dickey-Fuller (ADF) test with a significance level of $\alpha = 0.05$. The test yielded a Dickey-Fuller Statistic of -17.276 and p-value = 0.01, which is less than $\alpha = 0.05$. These results indicate that the time series data is stationary in mean. The p and q orders in the ARIMA model were identified from the plots of the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) values. The ACF and PACF plots are shown in Figure 2.

Based on Figure2(a), the ACF value is significant at lags 2, 6, and 10, while in Figure 2(b), the PACF value is significant at lags 2 and 10. The ACF and PACF value patterns do not show a gradual decline and are only significant at certain lags. Based on the patterns observed in the ACF and PACF, the restricted ARIMA model is considered appropriate. This model emphasizes only one or a few lags, determined by the significance of the ACF and PACF values at specific lags. The use of restricted ARIMA models can avoid excessive parameter complexity, making the parameter estimation process more stable. Based on the significant lag in ACF and PACF values, the proposed ARIMA models are ARIMA ([2],0,0), ARIMA (0,0,[2]), and ARIMA (0,0,[6]). The choice of ARIMA ([2],0,0) corresponds to the significant PACF at lag 2, whereas ARIMA (0,0,[2]) and ARIMA (0,0,[6]) are based on significant ACF values at lags 2 and 6. The tentative ARIMA model parameters were estimated via Maximum Likelihood Estimation (MLE) and the results of the parameter estimation are shown in Table 1.

In Table 1, the AR and MA components in all ARIMA models are significant., but the constant parameter $\hat{\mu}$ is not significant. After estimating the ARIMA model parameters, the next step is to test residual autocorrelation and residual normality, and choosing the best ARIMA model based on AIC value. The results of the autocorrelation and normality tests of the residuals and the AIC values of all ARIMA models are shown in Table 2.

In Table 2, the p-value from the residual autocorrelation test is above α=0.05, so it can be stated that there is no residual autocorrelation. However, the p-value from the residual normality test is below α=0.05, so the residuals are not normally distributed. These findings align with Tsay (2010), who noted that the assumption of normality in financial data is often violated. Nevertheless, according to Hyndman and Athanasopoulos (2018), time series models may still be used even

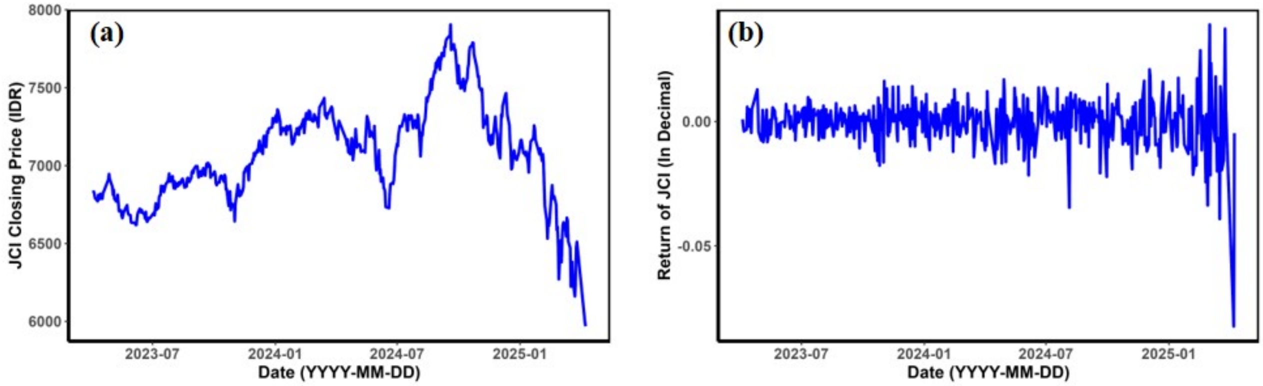

Figure 1. Time Series Plots of JCI and JCI Returns from April 2023 to April 2025: (a) JCI; (b) JCI Return Series Over the Same Period

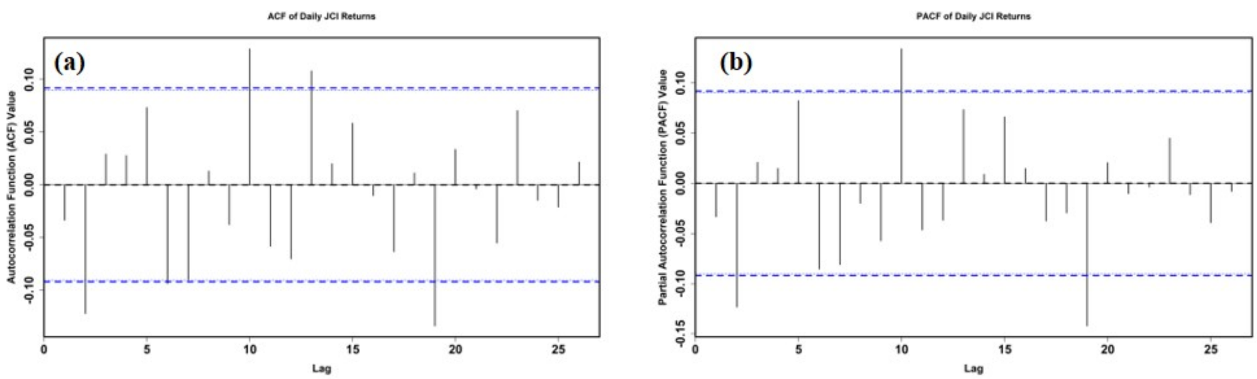

Figure 2. ACF and PACF Plot in JCI Returns: (a) Autocorrelation Function (ACF) and (b) Partial Autocorrelation Function (PACF) of the JCI Return Series.

Table 1. Estimated Parameters Values and Significance Tests for ARIMA Models in JCI Return

Model	Estimate Value	Z-Value	p-Value	Significance Status
ARIMA ([2],0,0)	$\hat{\phi}_2 = -0.144585$	-2.9159	0.003547	Significant
	$\hat{\mu} = -0.000261$	-0.6874	0.491834	Not Significant
ARIMA (0,0,[2])	$\hat{\theta}_2 = -0.13762$	-2.8796	0.003982	Significant
	$\hat{\mu} = -0.00026015$	-0.6951	0.486993	Not Significant
ARIMA (0,0,[6])	$\hat{\theta}_6 = -0.137862$	-2.5586	0.01051	Significant
	$\hat{\mu} = -0.0002688$	-0.7144	0.47501	Not Significant

Table 2. Results of Autocorrelation and Normality Tests of ARIMA Model Residuals and AIC Values

	Autocorrelation Test		Normality Test		
Model	Q-Ljung Box Value	p–Value	Jarque Bera Value	p–Value	AIC Value
ARIMA ([2],0,0)	0.63521	0.4254	2411.3	$< 2.2 \times 10^{-16}$	-3070.819
ARIMA (0,0,[2])	0.56043	0.4541	2486.3	$< 2.2 \times 10^{-16}$	-3070.506
ARIMA (0,0,[6])	0.65322	0.4190	3133.9	$< 2.2 \times 10^{-16}$	-3068.613

when the normality assumption is not fully satisfied. According to the AIC values in Table 2, the ARIMA ([2],0,0) model is the lowest AIC, so this model is selected for volatility modeling using the GARCH model.

3.2 GARCH Modelling

The ARCH effect was tested using the Lagrange Multiplier (LM) test with α=0.05. The Lagrange Multiplier test results showed an LM statistic of 8.112 and p-value=2,2×10^{-16}, which means there is an ARCH effect on the residuals, requiring a GARCH model. The GARCH model used is the GARCH (1,1) model. Table 3 shows the results of the GARCH (1,1) model parameter estimation.

In Table 3, the p-values for the parameters α_1 and β_1 are less than $\alpha = 0.05$, but for the parameter ω, they are above 0.05. The sum of α_1 and β_1 being equal to one reflects persistent volatility. These results are consistent with the JCI return data pattern, which shows increasing fluctuations at the end of the period. Based on the GARCH (1,1) parameter estimates, it is necessary to proceed with IGARCH (1,1) and FIGARCH (1,1) modeling. Table 4 shows the estimated values and significance tests for the IGARCH (1,1) and FIGARCH (1,1) model parameters.

In Table 4, the p-values for the parameter α_1 in the IGARCH(1,1) model are less than $\alpha = 0.05$, but in the FIGARCH (1,1) model they are above $\alpha = 0.05$. For the parameter ω in the IGARCH (1,1) model, the p-value is above 0.05, and for the parameter β_1 in the FIGARCH (1,1) model the p-value is less than $\alpha = 0.05$, but in the IGARCH (1,1) model there is no p-value, i.e., NA. The NA value for the β_1 parameter is caused by the standard error of β_1 being NA, which affects the t-test calculation and the resulting p-value. The estimated value for the parameter δ in the FIGARCH (1,1) model is close to 1, which means that there is no fractional long memory, but rather full integration, so that the FIGARCH (1,1) model can be reduced to the IGARCH (1,1) model. The standard error of β_1 being NA is caused by the dependence between parameters in the IGARCH (1,1) model, namely the β_1 and α_1 parameters. The form of this parameter dependency is $\beta_1 = 1 - \alpha_1$. As a result, the β_1 value is no longer an independent parameter, so the standard error value cannot be calculated. The IGARCH (1,1) model parameters presented in Table 4 were estimated using the Maximum Likelihood Estimation (MLE) method, which identifies parameter values that maximize the likelihood function based on the observed data. In order for MLE to provide standard errors and statistical inferences, each parameter must be independent and the variance of the estimator must be calculable (Greene, 2018). However, due to parameter dependencies in the GARCH (1,1) model, the variance of certain estimators cannot be calculated. Therefore, an approach suited to the model's characteristics, namely the Bayesian method, is required. The result of the Bayesian approach is a posterior distribution so that all estimators have distributions that are either interdependent or independent. However, prior to applying the Bayesian approach to GARCH modeling, it is essential to examine the asymmetry effects in both the GARCH (1,1) and IGARCH (1,1) models. This needs to be done because stock data generally has an asymmetry effect (Azimova, 2022; Ponziani, 2022; Chancharat and Chancharat, 2024). The asymmetric effect test on the IGARCH (1,1) model is presented in Table 5.

Table 5 shows that there is no sign bias, either negative or positive, in the GARCH (1,1), IGARCH (1,1) and FIGARCH (1,1) models. This means that there is no leverage effect on the data, but it is still necessary to perform GJR-GARCH and EGARCH modeling to compare with the IGARCH, GARCH, and FIGARCH models. Table 6 shows the estimated values and significance tests from the IGARCH (1,1) and FIGARCH (1,1) models parameters.

In Table 6, the p-value for parameter γ in the EGARCH and GJR–GARCH models is less than 0.05, so it can be stated that there is an asymmetric effect. From the results of GARCH (1,1), IGARCH (1,1), FIGARCH (1,1), EGARCH (1,1), and GJR-GARCH (1,1) modeling using estimation methods, one model will be selected based on the AIC value for GARCH modeling using the Bayesian approach. The AIC values for the GARCH model are shown in Table 3, for the IGARCH and FIGARCH models in Table 4, and for the EGARCH and GJR-GARCH models in Table 6. Among the five types of GARCH models, namely GARCH (1,1), IGARCH (1,1), FIGARCH (1,1), EGARCH (1,1), and GJR-GARCH (1,1), the IGARCH (1,1) model produced the smallest AIC value, as shown in Table 4. Therefore, IGARCH (1,1) is selected as the model for Bayesian GARCH modeling.

3.3 GARCH and IGARCH Bayesian

The Bayesian model was applied not only to the IGARCH model, which had the lowest AIC value, but also to the GARCH model. Although IGARCH was chosen as the most appropriate model, the GARCH model was still included because the parameter estimation results with MLE were very similar to the IGARCH parameter estimation, indicating comparable volatility. Bayesian parameter estimation for the IGARCH and GARCH models begins with the specification of prior distributions, and the prior distribution specified for all parameters is an independent Gamma distribution as defined in Equation (13). The Gamma distribution was chosen as the prior based on its suitability for modeling parameters related to variance, as explained in subsection 3.2.3. In the IGARCH (1,1) model, α_1 is defined as $1 - \beta_1$, with priors $\omega \sim \text{Gamma}(9, 10)$ and $\beta_1 \sim \text{Gamma}(0.45, 0.5)$. In the GARCH (1,1) model, $\omega \sim \text{Gamma}(9, 10)$, $\beta_1 \sim \text{Gamma}(0.45, 0.5)$, and $\alpha_1 \sim \text{Gamma}(0.05, 0.5)$. Posterior distributions were estimated using the Hamiltonian Monte Carlo (HMC) with leapfrog iteration using Equations (17), (18), (19) and (20) and also using the Metropolis Hastings (MH) algorithm to compare the convergence of the two algorithms. Sampling used 10,000 iterations with 1000 burn-in and thinning of 10. Convergence was assessed using trace, ergodic mean, kernel density, and ACF plots, as shown in Figure 3, Figure 4, Figure 5, and Figure 6.

Figure 3(a) and Figure 3(b), the trace plots for the IGARCH (1,1) and GARCH (1,1) model parameters generated by HMC display random fluctuations, indicating satisfactory posterior exploration. In contrast, the trace plots in Figure 3(c) and Figure 3(d), shows that fluctuations in sampling values in the Markov chain across all IGARCH (1,1) and GARCH (1,1) model parameters tend to move toward a certain value and do not spread evenly, suggesting limited posterior exploration.

Table 3. Estimated Parameters Value and Significance Tests of the GARCH (1,1) Model Using MLE Method

Model	Parameters	Estimate Value	t–Value	p–Value	AIC Value
GARCH(1,1)	ω	0.000001	0.26791	0.788769	-6.8418
	α_1	0.09	2.04316	0.041037	
	β_1	0.91	19.34645	0	

Table 4. Estimated Parameters and Significance Tests of the IGARCH (1,1) and FIGARCH (1,1) Model Using MLE Method

Model	Parameters	Estimate Value	t–Value	p–Value	AIC Value
IGARCH (1,1)	ω	0.000001	0.25553	0.79846	-6.8464
	α_1	0.09	1.97174	0.04864	
	β_1	1	NA	NA	
FIGARCH (1,1)	ω	0.000001	2.56723	0.01024	-6.834
	d	0.19	1.97174	0.098000	
	β_1	0.99999	57.87524	0	

Table 5. Sign Bias Test for the IGARCH (1,1) and GARCH (1,1) Models

Model	Sign Bias	t–Value	p–Value	Sign Bias Status
GARCH (1,1)	Negative Sign Bias	0.5399	0.6216	No Sign Bias
	Positive Sign Bias	0.7314	0.4649	No Negative Sign Bias
	Sign Bias	0.4883	0.6265	No Positive Sign Bias
IGARCH (1,1)	Negative Sign Bias	0.5414	0.5885	No Sign Bias
	Positive Sign Bias	0.7399	0.4597	No Negative Sign Bias
	Sign Bias	0.4853	0.6277	No Positive Sign Bias
FIGARCH (1,1)	Negative Sign Bias	0.5421	0.5880	No Sign Bias
	Positive Sign Bias	0.7378	0.4610	No Negative Sign Bias
	Sign Bias	0.4853	0.6277	No Positive Sign Bias

Table 6. Estimated Parameters and Significance Tests of the EGARCH (1,1) and GJR-GARCH (1,1) Model Using MLE Method

Model	Parameters	Estimate Value	t–Value	p–Value	AIC Value
EGARCH (1,1)	ω	0.00651	1.67225	0.094456	-6.846
	α_1	-0.0629	-3.7323	0.0019	
	β_1	0.99974	1499.59	0	
	γ	0.12969	10.4701	0	
GJR-GARCH (1,1)	ω	0.00763	0.2775	0.779861	-6.8459
	α_1	0.94452	43.9755	0	
	β_1	0.0937	4.49575	0.000007	
	γ	0.0937	4.49575	0.000007	

The ergodic mean line in Figure 4(a) and Figure 4(b), which are the ergodic means for IGARCH (1,1) and GARCH (1,1) obtained using the HMC algorithm, are able to reach a stable value quickly and this value is consistent throughout the iterations. In Figure 4(c) and Figure 4(d), the ergodic means obtained with the MH algorithm, are slower than the HMC algorithm in reaching stable values with greater initial fluctuations.

The kernel density plots in Figure 5(a) and Figure 5(b), which are the sampling parameters for IGARCH (1,1) and GARCH (1,1) obtained using the HMC algorithm show that all parameters form a smooth, unimodal density curve. The kernel density plots in Figure 5(c) and Figure 5(d), generated from Metropolis–Hastings (MH) sampling, also exhibit a unimodal distribution, although the curves appear less smooth than those obtained via HMC. Figure 6(a) and Figure 6(b) illustrate that the autocorrelation function (ACF) of the posterior samples for IGARCH (1,1) and GARCH (1,1) parameters obtained via the HMC algorithm exhibits negligible autocorrelation at the initial lags. However, ACF plots in Figure 6(c) and Figure 6(d), show that there are very high autocorrelation in the first few lags. Based on the trace plots, ergodic means, kernel density

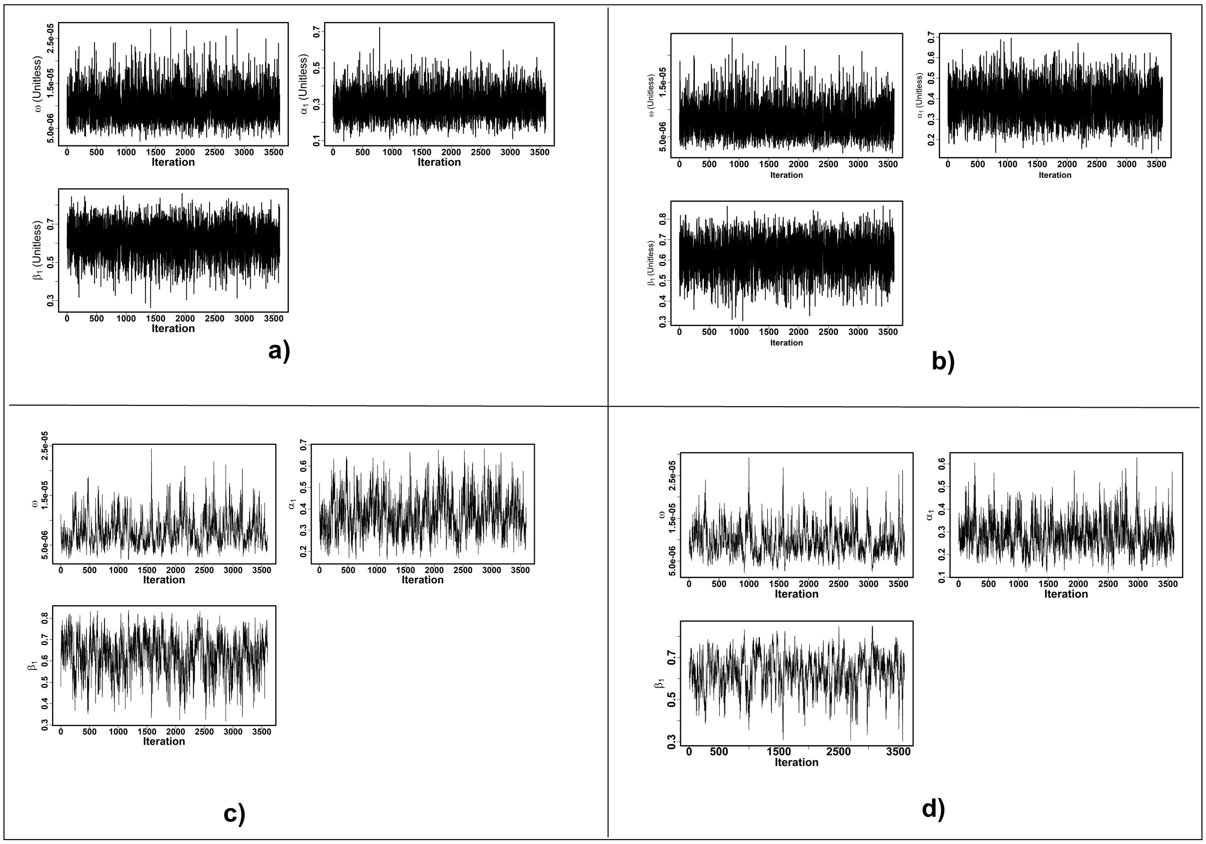

Figure 3. Trace Plots for the IGARCH(1,1) and GARCH (1,1) Models Parameters Using HMC and MH Algorithm: (a) IGARCH (1,1)-HMC, (b) GARCH (1,1)-HMC, (c) IGARCH (1,1)-MH, (d) GARCH (1,1)-MH

estimates, and ACF plots, the HMC algorithm demonstrates superior performance compared to the Metropolis–Hastings algorithm in exploring the posterior distribution. In addition to visual convergence evaluation, evaluation was also conducted using numeric methods such as Monte Carlo errors, Gelman-Rubin statistics, and effective sample sizea and the results are presented in Table 7.

In Table 7, the $\hat{R}$ values across all parameters in the IGARCH (1,1) and GARCH (1,1) models with the HMC algorithm is equal to 1, while for the MH algorithm it is equal to 1.01. Based on $\hat{R}$ values, the parameter samples for the IGARCH (1,1) and GARCH (1,1) models obtained via both the HMC and MH algorithms can be considered converged. However, the MC error values in the IGARCH (1,1) and GARCH (1,1) models with the HMC algorithm are smaller than those with the MH algorithm, and Figure 7 shows that the effective sample size (ESS) values for the IGARCH (1,1) and GARCH (1,1) models obtained using the HMC algorithm are substantially higher than those from the MH algorithm. Based on both graphical and numerical convergence results, the HMC algorithm demonstrates superior performance over the MH algorithm in exploring the posterior distributions of IGARCH (1,1) and GARCH (1,1) model parameters., aligning with findings by Liang et al. (2024) that HMC is generally more effective for estimating parameters in GARCH models. After confirming convergence, posterior inference was conducted by calculating the posterior means along with their 95% credible intervals. The results are presented in Table 8.

As shown in Table 8, parameter estimate values in the IGARCH (1,1) and GARCH (1,1) models obtained using Bayesian HMC estimation are not different from the estimates obtained using the MH algorithm. Furthermore, all parameters of the IGARCH (1,1) and GARCH (1,1) models, estimated using both HMC and MH algorithms, are statistically significant. The significance of IGARCH (1,1) and GARCH (1,1) model parameters estimated using Bayesian methods with HMC and MH algorithms differs from that obtained via MLE, as shown in Table 3 and Table 4. However, the values of the parameter estimators α_1 and β_1 between the IGARCH (1,1) and GARCH (1,1) models differ from the MLE method; in the Bayesian approach, the values of the parameter estimators α_1 and β_1 in the GARCH (1,1) model are not equal to one.

In the IGARCH (1,1) and GARCH (1,1) models estimated with the MLE approach, the ω parameter is not statistically significant, although its estimated value remains close to zero. Although a non-zero prior was specified for the Bayesian

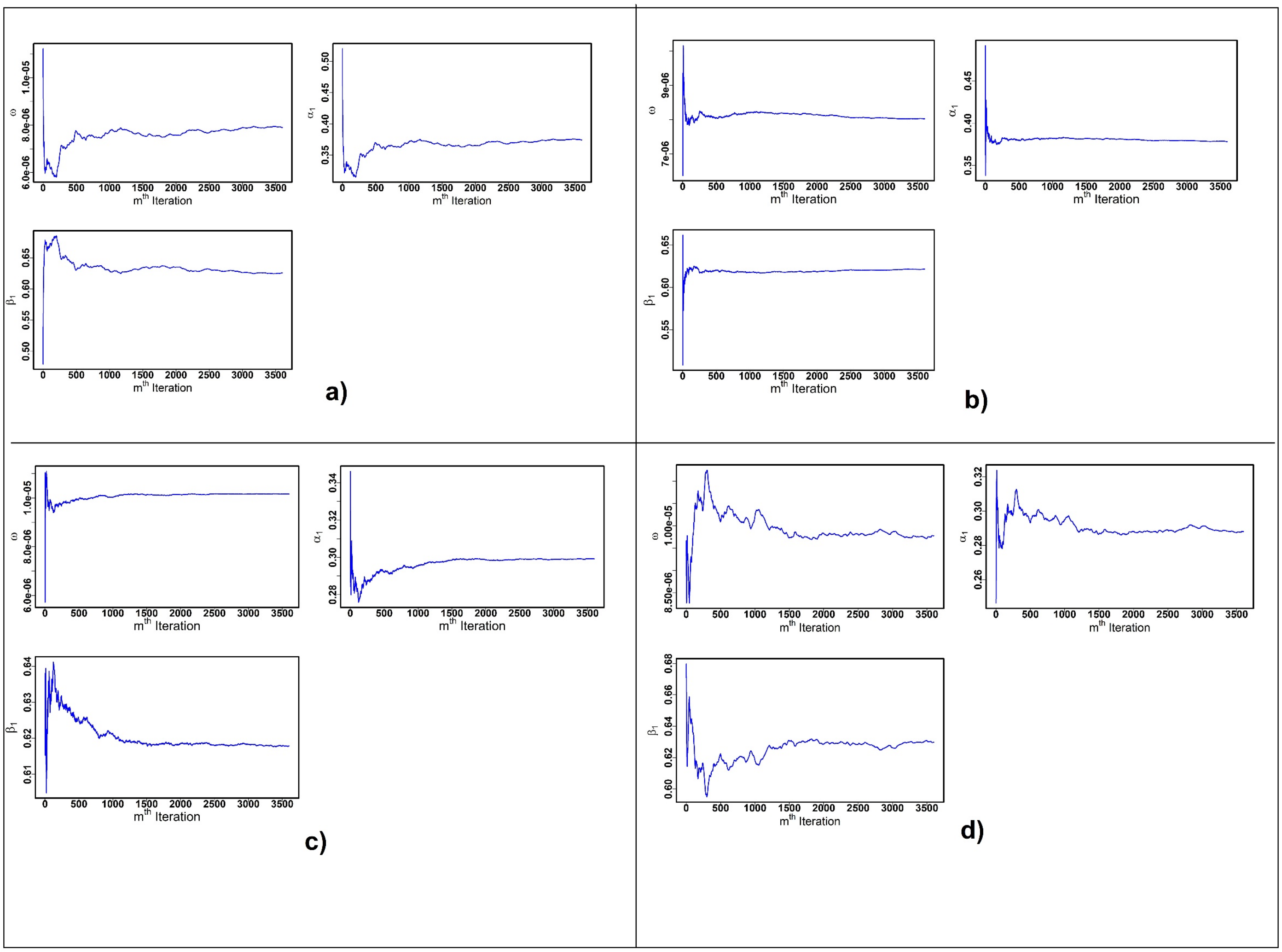

Figure 4. Ergodic Mean Plots for the IGARCH (1,1) and GARCH (1,1) Models Parameters Using HMC and MH Algorithm: (a) IGARCH (1,1)-HMC, (b) GARCH (1,1)-HMC, (c) IGARCH (1,1)-MH, (d) GARCH (1,1)-MH

IGARCH (1,1) and GARCH (1,1) models estimated via HMC, the posterior mean for the ω parameter remains close to zero and statistically insignificant. This result occurs because the likelihood function obtained from the data is stronger than the prior distribution in estimating the posterior parameters of ω. For the β_1 parameter in the IGARCH (1,1) model, significance cannot be assessed under MLE due to the standard error being NA. In contrast, the Bayesian IGARCH (1,1) model using HMC and MH algorithms allows the significance of β_1 to be evaluated. After testing the significance of the parameters, the suitability of the model is evaluated through graphical posterior predictive checks and numerical evaluation using posterior predictive values, and comparing the IGARCH-Bayesian HMC, GARCH-Bayesian HMC, IGARCH-Bayesian MH, and GARCH-Bayesian MH models using WAIC values. The result of the posterior predictive check is shown in Figure 8, and the posterior predictive values and WAIC values are shown in Table 9.

Figure 8(a) and Figure 8(c) show that the average value of the ARIMA ([2],0,0) model residual squares is at the left of the center of the IGARCH (1,1) model posterior distribution obtained using the Bayesian approach with the HMC and MH algorithms. However, in the GARCH (1,1) and IGARCH (1,1) models that use the Bayesian HMC and MH methods, as shown in Figure 8(b) and Figure 8(d), the central value of the actual data aligns closely with the posterior distribution center. This means that most of the variance values generated by the IGARCH (1,1) model using Bayesian HMC and MH exceed the observed data, while the variance values from the GARCH (1,1) model with Bayesian HMC and MH strictly match the observed data, indicating that both GARCH (1,1) models provide good fit. Nevertheless, the IGARCH (1,1)-Bayesian HMC and IGARCH (1,1)-Bayesian MH models remain adequate because the mean of the residual squares is still

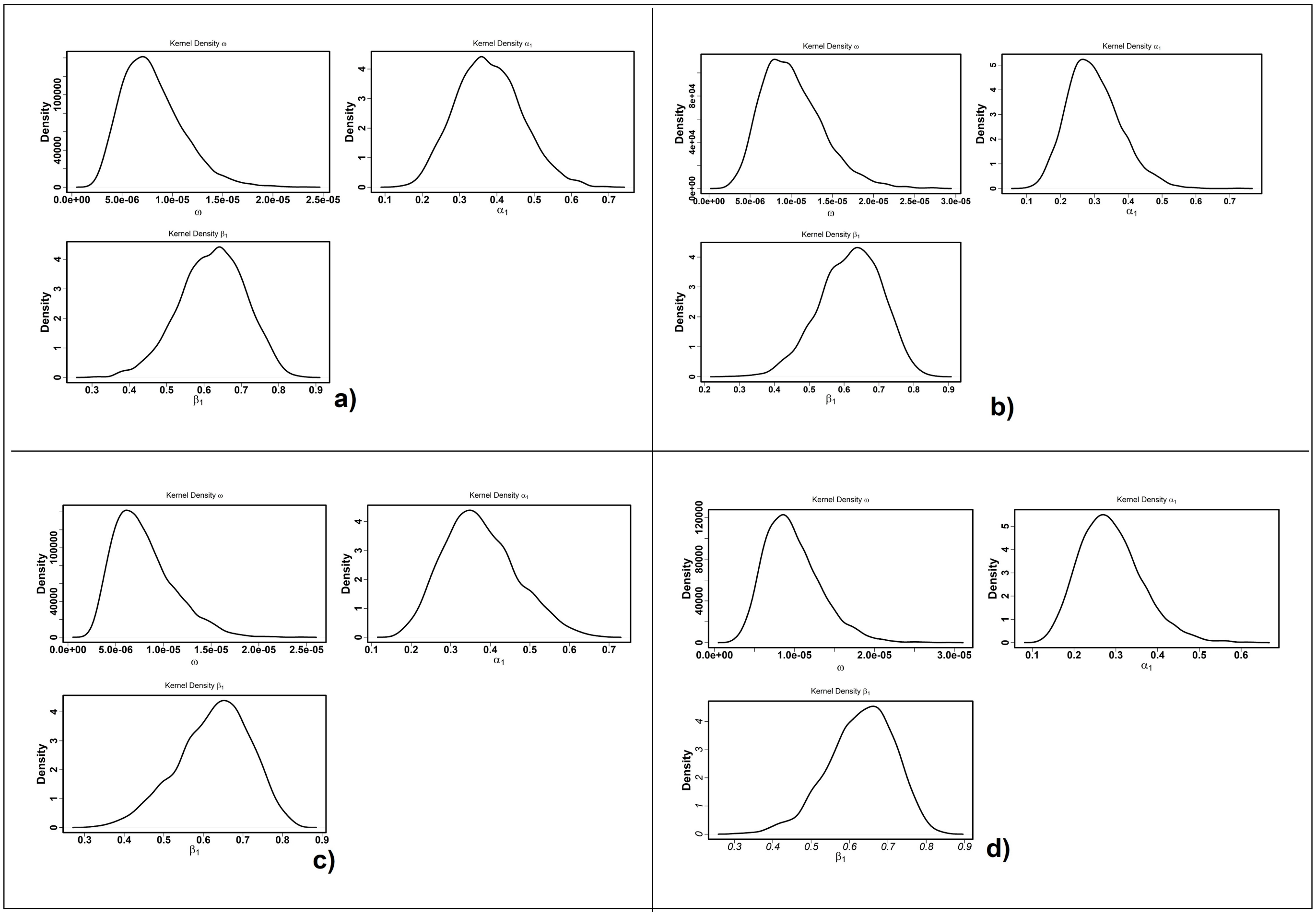

Figure 5. Kernel Density Plots for the IGARCH(1,1) and GARCH (1,1) Models Parameters Using HMC and MH Algorithm: (a) IGARCH (1,1)-HMC, (b) GARCH (1,1)-HMC, (c) IGARCH (1,1)-MH, (d) GARCH (1,1)-MH

Table 7. Numerical Convergence Diagnostics Using Gelman-Rubin Statistic, Effective Sample Size, and MC Error Value

Model	Parameter	Gelman-Rubin Statistic	Effective Sample Size	MC Error Value
IGARCH (1,1)-Bayesian HMC Algorithm	ω	1	3619	0.000000
	α_1	1	3452	0.000026718
	β_1	1	3452	0.000026718
GARCH (1,1)-Bayesian HMC Algorithm	ω	1	3442	0.000000
	α_1	1	3438	0.0000232693
	β_1	1	3389	0.0000265565
IGARCH (1,1)-Bayesian MH Algorithm	ω	1.01	233.4843	0.00000013346
	α_1	1.01	270.3463	0.0003380479
	β_1	1.01	270.3463	0.0003380479
GARCH (1,1)-Bayesian MH Algorithm	ω	1.01	210.8254	0.00000016648
	α_1	1.01	314.8322	0.0002358399
	β_1	1.01	191.5442	0.000449609

within the simulated distribution range of the IGARCH (1,1) model. The Posterior Predictive for IGARCH (1,1)-Bayesian HMC and IGARCH (1,1)-Bayesian MH shown in Table 9, indicating that the model tends to overestimate, but this value

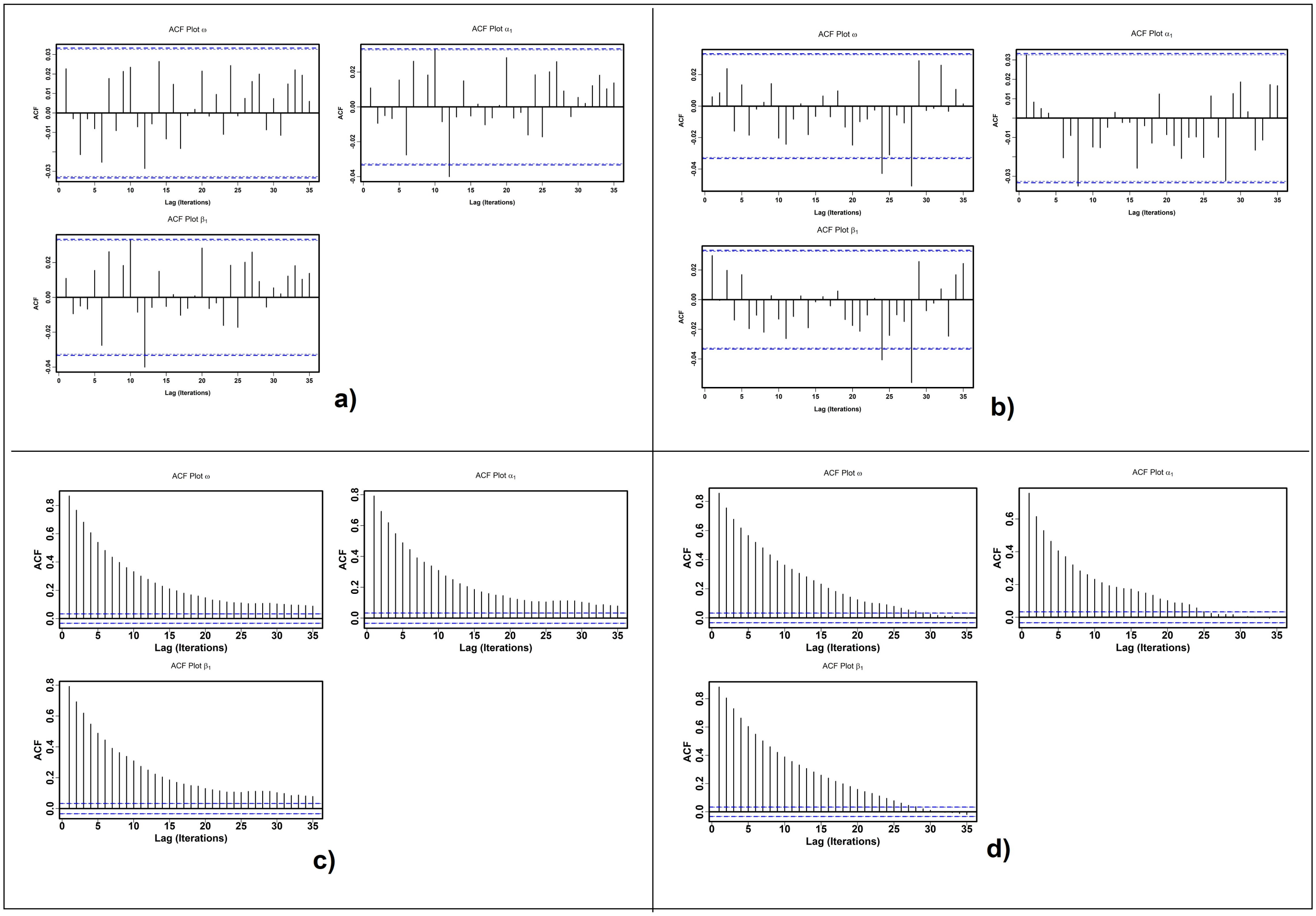

Figure 6. ACF Plots for the IGARCH (1,1) and GARCH (1,1) Models Parameters Using Bayesian HMC and MH Algorithm: (a) IGARCH (1,1)-Bayesian HMC, (b) GARCH (1,1)-Bayesian HMC, (c) IGARCH (1,1)-Bayesian MH, (d) GARCH (1,1)-Bayesian MH

falls within an acceptable range, and thus the IGARCH (1,1)-Bayesian HMC and IGARCH (1,1)-Bayesian MH model can be considered a good fit. Based on the results of the posterior predictive check and posterior predictive value, it can be concluded that all four models are good models. However, the best model is selected by looking at the smallest WAIC value. The IGARCH (1,1) model with a Bayesian approach using the MH algorithm is the model with the smallest WAIC value compared to other Bayesian GARCH models, as shown in Table 9. However, the IGARCH (1,1) model estimated using Bayesian approach with MH algorithm has a low ESS of approximately 200, so it is necessary to consider another Bayesian GARCH model, namely the IGARCH (1,1) model estimated using the Bayesian HMC algorithm approach with the second lowest WAIC value. The effective sample size produced in this model is 3000, which is much larger than the IGARCH (1,1) model with the Bayesian approach using MH algorithm. Thus, the IGARCH (1,1) model estimated using Bayesian parameter estimation with the HMC algorithm is the best model.

The estimated values of the IGARCH (1,1) model parameters obtained with the Bayesian parameter estimation method using the HMC algorithm are $\omega = 0.000$, $\alpha_1 = 0.38$, and $\beta_1 = 0.62$. A value of ω close to zero means that the variance of the Jakarta Composite Index (JCI) is truly influenced by past shocks and previous volatility, rather than by a constant component (ω). The coefficient α_1 of 0.38 indicates that the JCI index value in the current period responds relatively quickly to new information or unexpected events from the previous period, while the coefficient β_1 of 0.62 indicates that the effect of volatility from the previous period on the current volatility level is moderate. Since $\alpha_1 + \beta_1 \approx 1$, this model confirms the integrated nature of volatility, meaning that shocks occurring at that time have a long-term effect on the JCI and do not decrease exponentially over time. Economically, this finding reflects the behavior of emerging markets, where investors tend to respond quickly to new information but take longer to restore stabil-

Table 8. Posterior Means, 95% Credible Intervals, and Significance Status for IGARCH(1,1) and GARCH(1,1) Models Parameters Using Bayesian HMC and MH Algorithms

Model	Parameter	Posterior Mean	2.5% Percentile	97.5% Percentile	Significance Status
IGARCH(1,1)-Bayesian HMC Algorithm	ω	0.000	0.00	0.00	Significant
	α_1	0.38	0.22	0.57	Significant
	β_1	0.62	0.43	0.78	Significant
GARCH(1,1)-Bayesian HMC Algorithm	ω	0.000	0.00	0.00	Significant
	α_1	0.30	0.17	0.43	Significant
	β_1	0.62	0.43	0.78	Significant
IGARCH(1,1)-Bayesian Metropolis Hastings Algorithm	ω	0.0000079	0.000003422	0.00001527	Significant
	α_1	0.2741313	0.2191	0.5699	Significant
	β_1	0.6258687	0.4301	0.7809	Significant
GARCH(1,1)-Bayesian Metropolis Hastings Algorithm	ω	0.000009776	0.000004285	0.00001794	Significant
	α_1	0.288	0.1642	0.24534	Significant
	β_1	0.6298	0.4391	0.7763	Significant

Table 9. Posterior Predictive Value and WAIC Value for IGARCH and GARCH Models Estimated Using Bayesian HMC and Bayesian MH

Model	Posterior Predictive Value	WAIC Value
IGARCH(1,1)-Bayesian HMC	0.8477778	-3219.391
GARCH(1,1)-Bayesian HMC	0.4877778	-3214.656
IGARCH(1,1)-Bayesian MH	0.8416667	-3219.499
GARCH(1,1)-Bayesian MH	0.4747222	-3215.994

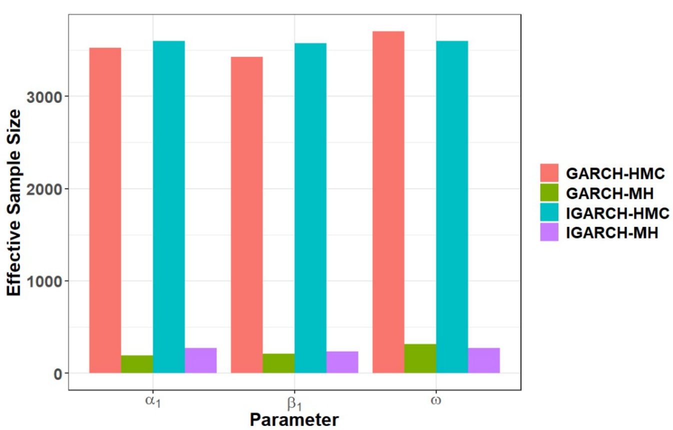

Figure 7. Comparison of Effective Sample Size on IGARCH (1,1) and GARCH (1,1) Models Parameters Using Bayesian HMC and MH Algorithms

ity. In developing countries such as Indonesia, stock market volatility is often influenced by external shocks such as global interest rate fluctuations, commodity prices, geopolitical risks, and even spillover effects from international markets. Recent empirical studies confirm the existence of such cross-market effects. In their research, Gunawan et al. (2022) found that external variables such as US interest rate movements and global index returns significantly affect the volatility of Indonesian ESG stocks. Research by Setiahutami and Chalid (2024) shows that the stock market in Indonesia is affected by fluctuations in world oil prices, palm oil prices, and currency exchange rates. The results of research conducted by Silva et al. (2025) show that there is significant two-way volatility spillover between the Indonesian, Malaysian, and Thai markets. Rachman et al. (2025) found that macroeconomic indicators, particularly global commodity prices and US monetary tightening, affect asymmetric volatility in the Indonesian stock market, with a stronger response to negative shocks. Lim et al. (2025) prove that there is a significant impact of global oil price volatility on the Indonesian stock market, especially during periods of geopolitical uncertainty. The impact of this volatility tends to be persistent and stronger during phases of declining oil prices.

3.4 Model Evaluation and Forecasting

To evaluate the model, can compute the fitted variances from the IGARCH (1,1) model based on the Bayesian estimates using Equations (27) and Equation (28). After that, calculate the difference between the IGARCH (1,1)-Bayesian HMC fitted values and the reliazed volatility. The fitted variances

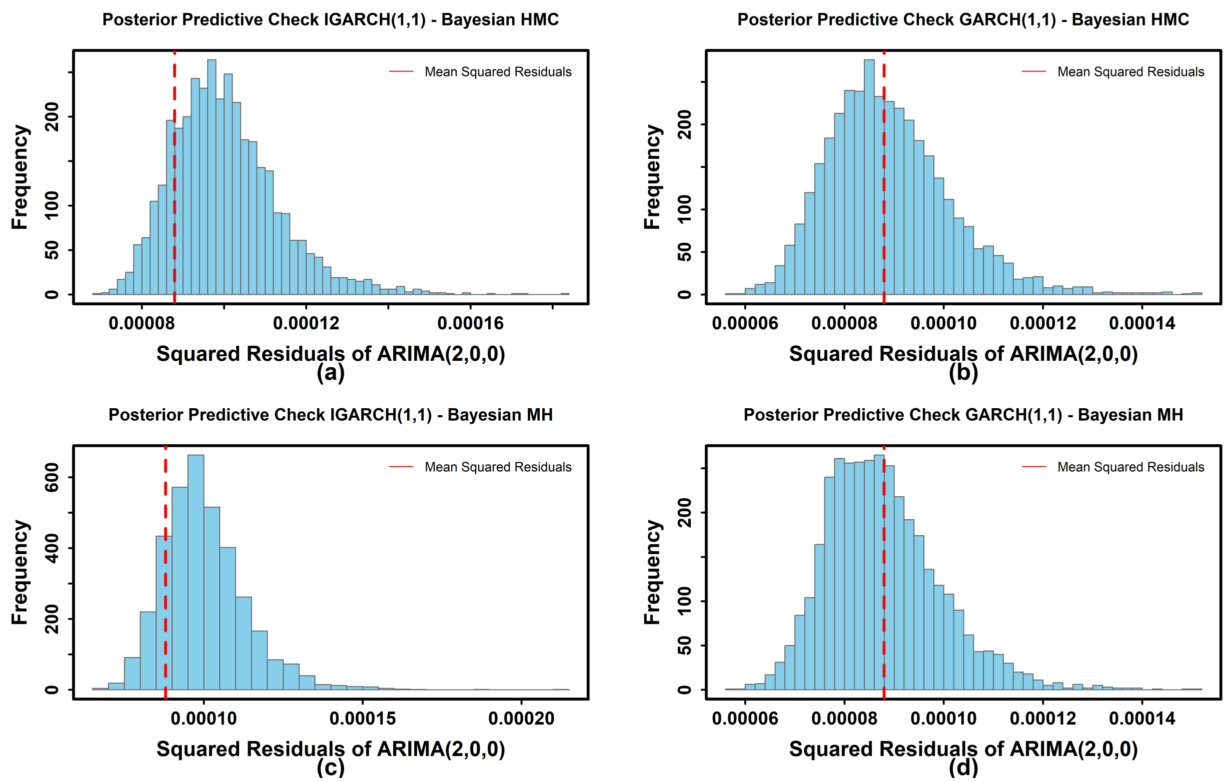

Figure 8. Posterior Predictive Check Comparing Squared Residuals of ARIMA ([2],0,0) with Simulated Variances from the IGARCH and GARCH Models With Bayesian Approach: (a) IGARCH (1,1)-Bayesian HMC, (b) GARCH (1,1)-Bayesian HMC, (c) IGARCH (1,1)-Bayesian MH, (d) GARCH (1,1)-Bayesian MH

from the IGARCH(1,1) model can be derived using Equation (28) and the realized volatility using Equation (30) (Zhao et al., 2024).

$$\hat{\sigma}_{t(\text{realized})} = \sqrt{\frac{1}{k-1}\sum_{i=0}^{k-1} e_{t-i}^2} \quad (30)$$

In Equation (30), $\hat{\sigma}_{t(\text{realized})}$ represents the actual volatility, e_{t-i}^2 denotes the squared residual at time $t - i$, and k is the number of lag windows used, which is generally set to $k = 5$. The graphical comparison between the fitted values of the IGARCH (1,1)-Bayesian HMC model and the actual volatility is presented in Figure 9.

In Figure 9, the volatility values obtained from the Bayesian HMC IGARCH (1,1) model fit show slight differences from the actual values in July 2023 and December 2024, but still follow the actual volatility pattern. From January 2025 to March 2025, the estimated values are exactly the same as the actual values. Based on a comparison between the values estimated using the model IGARCH (1,1) and the actual volatility values, it can be concluded that the IGARCH (1,1) model obtained with Bayesian approach using the HMC algorithm is capable of modeling JCI volatility well. In addition to evaluating the fitted

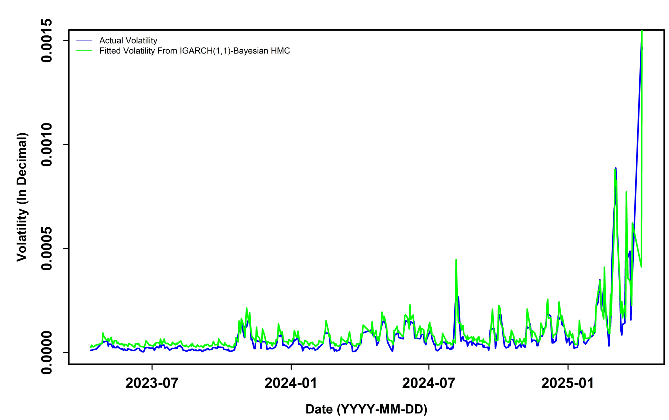

Figure 9. Comparison Between Actual Volatility and Fitted Values from IGARCH (1,1) Bayesian HMC Model

variance value with the actual variance, the fitted return value is also evaluated with the actual JCI return value. The fitted value of the IGARCH (1,1) Bayesian HMC model for return data were obtained by returning the residuals that had been modeled using the IGARCH (1,1) Bayesian HMC model to the average model, namely the ARIMA ([2],0,0) model. The fitted return value of the ARIMA ([2],0,0)-IGARCH (1,1) Bayesian HMC model is obtained through Equation (31).

$$r_t = -0.000261 - 0.144585\, r_{t-2} + e_t \quad (31)$$

where $e_t = z_t\sqrt{\hat{\sigma}_t^2}$, $z_t = \frac{e_t}{\sqrt{\hat{\sigma}_t^2}}$, and $\hat{\sigma}_t^2$ is obtained from Equation (28). The results of the IGARCH(1,1) Bayesian HMC model evaluation, obtained by reintegrating it into the mean model, are shown in Figure 10 and Table 10.

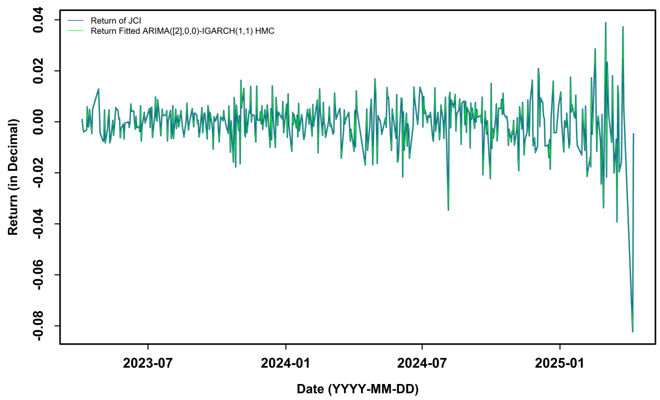

Figure 10. Comparison of Actual JCI Returns and Fitted Values from the ARIMA ([2],0,0)-IGARCH (1,1) Bayesian HMC Model

Table 10. Mean Absolute Error (MAE) and Relative Error Values from Fitted ARIMA ([2],0,0)-IGARCH (1,1)-Bayesian HMC Model on Variance and Return Series

Data	MAE Value	Relative Error
Variance	0.0000354367	0.02380543
Return	7.53×10^{-20}	6.2092×10^{-19}

Figure 10 shows that the return values of the Jakarta Composite Index (JCI), as fitted by the ARIMA ([2],0,0)-IGARCH (1,1) model with the Bayesian HMC approach, can closely follow the actual return patterns. Based on the Mean Absolute Error (MAE) and Relative Error values on variance, it shows that the IGARCH (1,1) model has good capabilities in modeling JCI volatility. The MAE value in the return section is very small because the return value scale is close to zero. Therefore, it is necessary to look at the relative error value obtained from comparing the MAE value and the range of JCI return data. The relative error value obtained is 6.2092×10^{-19} which means that the error is 6.2092×10^{-19}% of the JCI return data range, so the error value is very small. Considering the MAE, Relative Error, and the comparison plot, it can be stated that the IGARCH (1,1) model with the Bayesian HMC algorithm has a high capability in capturing the volatility dynamics of the JCI, making it a reliable model for forcasting. Variance forecast is calculated using multi-step recursive method because this study forecasts JCI variance over a period of 100 periods. Forecasting volatility using IGARCH (1,1)-Bayesian HMC is shown in Figure 11.

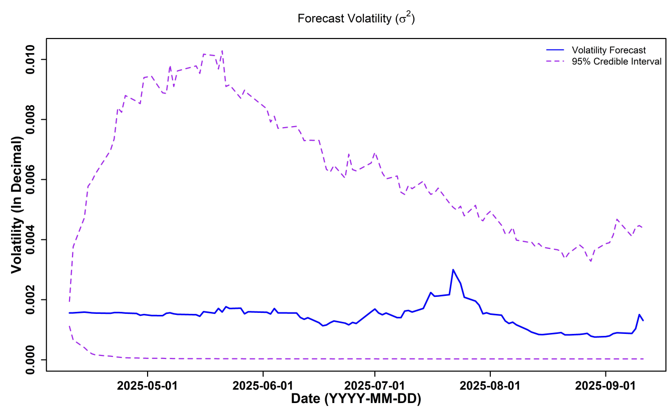

Figure 11. Multi-Step-Ahead Volatility Forecasts of JCI Using the IGARCH(1,1) Bayesian HMC Model and 95% Credible Intervals of Volatility Forecast

Figure 11 shows that the point forecasts of volatility are stable and fall within a relatively low range, approximately 0.001 to 0.002. This indicates that the Jakarta Composite Index (JCI) will experience stable volatility over the next 100 periods. In mid-April 2025, JCI volatility had declined again and remained stable until September 2025. However, the credible intervals suggest a potential for elevated volatility, particularly in the early stages of the forecast horizon. The intervals widen significantly and peak around the end of May, then gradually narrow in subsequent periods. This indicates that volatility was higher in the early forecast period but decreased over time, demonstrating greater confidence in the model's long-term forecasts. From an economic perspective, this pattern reflects how markets adjust to new information. Within the IGARCH (1,1) framework, the estimated parameters generally show a small α_1 value (sensitivity to shocks) and a large β_1 value (persistence level), which means that even though markets respond moderately to new shocks, the effects of these shocks tend to diminish and return to stability over time. This type of behavior is typical of emerging markets such as Indonesia, where investor sentiment and information dissemination occur gradually. The slow decline in volatility indicates that market participants tend to adjust their expectations cautiously after the government policy is announced, resulting in prolonged volatility.

Widening confidence intervals at the beginning of the forecasting period indicate increased short-term uncertainty, triggered by external macroeconomic events or government policy announcements. However, as the forecast horizon extends and the interval narrows, it indicates that the market is gradually regaining stability. The results of the Credible Interval show that the Indonesian stock market maintains short-term vulnerability but has medium-term resilience. Economically, these findings indicate that even though average market conditions appear stable, investors must remain vigilant against potential shocks that could have a long-term impact on volatility due to their persistent nature. For policymakers, the initial expansion of the credible interval serves as an early warning indicator of short-term financial vulnerability, highlighting the importance of transparent communication and timely macroprudential measures to maintain market confidence.

4. CONCLUSIONS

The Jakarta Composite Index (JCI) data have persistent volatility because the sum of $\hat{\alpha}_1$ and $\hat{\beta}_1$ from the GARCH (1,1) model is equal to one. These results are consistent with the JCI return data pattern, which shows increasingly high fluctuations at the end of the observation period. The application of the Bayesian IGARCH (1,1) using the HMC algorithm was able to capture the volatility of JCI. Forecast results indicate that the JCI is expected to show a relatively stable volatility pattern over the next 100 periods. However, credible intervals reveal uncertainty and the potential for sudden volatility spikes, which are crucial for informed risk-aware investment decisions. This research contributes significantly to the field by providing a rigorous and scalable Bayesian framework for volatility modeling in emerging markets, where traditional frequentist approaches often face limitations due to data irregularities and market inefficiencies. The use of the Bayesian paradigm, coupled with the HMC algorithm, enhances the interpretability, transparency, and robustness of the inferential process, enabling more reliable conclusions about financial risk. The results of this study are highly relevant for investors and financial authorities in emerging markets such as Indonesia, where political and macroeconomic uncertainty often triggers sudden market responses. Investors need to establish a great risk management strategy to avoid significant losses. For financial authorities, it is important to optimize stock price control policies because stock prices fluctuate greatly and are easily influenced by social and political issues. Given the promising results, several directions are proposed for future research. These include incorporating exogenous macroeconomic variables (e.g., IGARCH-X or Bayesian VAR-GARCH) or global factors because the volatility of JCI depends on macroeconomic variables and global factors.

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