

Cement Compressive Strength Control Using CUSUM and MCUSUM Control Chart

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Abstract

Compressive strength is one of the test factors used to determine whether cement production is in a controlled state or not. Portland type composite cement or PCC is the cement that is widely used in infrastructure development. The Cement of 3-days compressive strength, 7-days compressive strength, and 28-days compressive strength are the variables that will be controlled in this study. The normal distribution test and correlation test show that the data on each variable is normally distributed, and each variable has a strong correlation. Univariate cement control using the cumulative sum control chart (CUSUM) and multivariate control using a multivariate cumulative sum (MCUSUM) control chart is performed to obtain the best control results. Correlated variables show that control using a multivariate control chart results in fewer out of control observations compared to a univariate control chart. This explains that the MCUSUM control chart is more sensitive than the CUSUM control chart in controlling observations of data out of control.

Keywords

Cement compressive strength, Control chart, CUSUM

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1. INTRODUCTION

Cement is one of the main basic ingredients of building construction, which is a strategic commodity. In 2019 will be a new chapter in the cement sales target in Indonesia, which estimates that cement sales have grown around 4-5%. This growth has taken into account inflation and the rupiah exchange rate against the United States dollar (US\$). The Indonesian cement group targets that in 2019 it can export 4 million tons of cement. Indonesian cement production has fulfilled this demand to obey the quality and price competitiveness so that the control over the quality of cement must be a concern.

Cement industry companies certainly have a target to prepare the highest-quality products. The company carries out several stages in maintaining the quality of its products. One way is to do a compressive strength test. There are three stages of compressive strength testing performed on cement production. They are namely compressive strength of 3 days, the compressive strength of 7 days, and compressive strength of 28 days. So, these are three compressive strength stages that will become variables in the compressive strength test of the portland composite cement in SNI15-7064-2004. One way to improve the quality of cement production is by controlling each statistical quality test in cement production.

One of the statistical quality control techniques for testing the compressive strength of cement is a quality control chart (control chart). Control charts are used to monitor product quality from a production process and can detect very small mean shifts. It needs to do a single and multivariate control of variables that affect the quality of a product. One of the most important tools in controlling the quality of production from one time to another for one variable is the cumulative sum control chart (CUSUM). While the tool used to control product quality for multivariate data is the multivariate cumulative sum (MCUSUM) control chart.

Woodall and Montgomery had discussed that CUSUM control charts can be used in a broad industrial process, due to the fact that online measurement and distributed computing systems are widely used in statistical process control (SPC) applications (Woodall and Montgomery, 1999). The CUSUM control chart is a control chart that can quickly detect small shifts that are being processed. This control chart is designed to detect processes that are continuously shifting by accumulating deviations from the target (Faisal et al., 2018). The CUSUM control chart is able to detect shifts in both mean and variance, and also identify points of change from these shifts (Khoo, 2004; Wu et al., 2007). Meanwhile, Wu et al. (2009) said that the CUSUM control chart could be used to estimate the size of the average shift by looking

at the statistical slope. Koshti (2011) in his research, also said that the CUSUM control chart is the most efficient chart compared to the Shewhart control chart. Ghomi and Sogandi (2019) discussed further control charts using a two-sided Bernoulli-based CUSUM control chart with autocorrelation observations. Prajapati (2015) has discussed the effectiveness of conventional CUSUM control charts for observational correlations, whereas Aminnayeri and Sogandi (2016) have explained CUSUM control charts more about Bernoulli CUSUM Self-starting, which starts from risk-adjusted to dynamic probability control limits.

A multivariate CUSUM approach discussed by Alwan has similarities to the familiar univariate CUSUM V-mask procedure (Alwan, 1986). MCUSUM control charts are widely used in industry because they are powerful and easy to use (Hamed et al., 2016). The MCUSUM chart is a multivariate extension of the cumulative sum chart. The multivariate CUSUM reduces to a univariate normal CUSUM (Healy, 1987). Multivariate CUSUM control chart, which is based on self-adaption of its reference value according to the information from current process readings, to quickly detect the multivariate process mean shifts (Dai et al., 2011). A multivariate cumulative sum (MCUSUM) control chart is one type of multivariate control chart for monitoring the mean vector (Nidsunkid et al., 2018). Mahmoud and Maravelakis (2013) explained the performance of multivariate CUSUM control charts with estimated parameters. Choi and Cho (2016) estimated the multivariate CUSUM control charts for monitoring the covariance matrix. Qiu and Hawkins (2001) have explained a rank-based multivariate CUSUM procedure. Woodall and Ncube (1985) in his research, also discussed multivariate CUSUM quality control procedures. Testik and Runge (2014); Golosnoy et al. (2009) estimated the multivariate CUSUM chart. Coronel-Brizio and Hernández-Montoya (2010) explained about normal distribution test, which used in this research, whereas Mukaka (2012) discussed the correlation coefficient of variables in this research. Dual Multivariate CUSUM Mean Charts also discussed by Haq et al. (2019). A comparison of Multivariate CUSUM Charts had been explained with Pignatiello Jr and Runger (1990). Midi and Shabbak (2011) has used the robust multivariate control chart to detect small shifts in mean. Sari and Devianto (2020) explain that multivariate exponentially weighted moving average control chart for monitoring exponentially distributed quality characteristics of products, where the exponential distribution is characterized by the characteristic function in the sense of convolution by Devianto (2016, 2018). The sum of an exponential distribution is the hypoexponential distribution can be set by the stabilizer constant such as describe by Devianto et al. (2015a,b).

Cement production has more than one test carried out so that the cement produced meets SNI standards and can compete. The Compressive strength test is a test that will be investigated whether controlled or not, which is one way of pulling information based on data mining of cement production. The compressive strength test consists of a 3-days compressive strength test, a 7-days compressive strength test, and a 28-days compressive strength test, which are continuous and related to each other. Because the compressive strengths have the small mean shift

in the process and it is time series data, therefore CUSUM control chart and MCUSUM control chart are chosen to find out whether each compressive strength test and the whole process of controlling the compressive strength test of PCC type cement is controlled or not.

2. EXPERIMENTAL SECTION

2.1 Materials

The material in this study is Portland type composite cement or PCC, with data are collected as the cement of 3-days compressive strength, 7-days compressive strength and 28-days compressive strength. The tool's analysis to control the compressive strength of cement used in this study is the CUSUM control chart and MCUSUM control chart.

2.1.1 CUSUM Control Chart

The CUSUM method uses two ways to monitor the production process. One of them is tabular CUSUM. Let x_i be the i -th observation in the production process. When the process is controlled, x_i is normally distributed with an average of μ_0 and a standard deviation of σ . The CUSUM tabular is obtained from collecting the calculation results from the μ_0 at the upper target using C+ and collecting the calculation results from the μ_0 at the top target using C-. Upper CUSUM and Lower CUSUM for i -observation, where $i = 1, \dots, n$ are (Hamed et al., 2016):

$$C_i^+ = \max[0, x_i - (\mu_0 + K) + C_{i-1}^+] \quad (1)$$

$$C_i^- = \max[0, (\mu_0 + K) - x_i + C_{i-1}^-] \quad (2)$$

where

$$C_0^+ = C_0^- = 0 \quad (3)$$

$$\mu_1 = \mu_0 + \sigma \quad (4)$$

$$K = \frac{\mu_1 + \mu_0}{2} \quad (5)$$

Upper Center Line (UCL) and Lower Center Line (LCL) in CUSUM control chart are

$$UCL = h\sigma, \text{ for } h = 4\sigma \quad (6)$$

$$LCL = h\sigma, \text{ for } h = -4\sigma - 5 \quad (7)$$

The process is said to be uncontrollable if $C_i^+ > UCL$ and $C_i^- < LCL$. For uncontrolled observations, determine the new process target values, namely:

$$\hat{\mu} = \mu_0 + K + \frac{C_i^+}{N^+}, \text{ for } C_i^+ > UCL \quad (8)$$

$$\hat{\mu} = \mu_0 - K - \frac{C_i^-}{N^-}, \text{for } C_i^- > UCL \tag{9}$$

where

N^+ :The number of consecutive periods of C_i^+ that is not zero.

N^- :The number of consecutive periods of C_i^- that is not zero.

2.1.2 MCUSUM Control Chart

The multivariate CUSUM (MCUSUM) control chart is generalized for CUSUM control chart. Let X_i be a random vector containing elements for the i -th observation, which derives from the p variate normal distribution with an in-control population mean vector μ_0 and an in-control common $p \times p$ covariance matrix, Σ_0 . The CUSUM for detecting a change in the variance-covariance matrix may be written as (Pignatiello Jr and Runger, 1990).

$$C_i = \max[D_i' \sum_0^{-1} D_i]^{-\frac{1}{2}} - K n_i, 0 \tag{10}$$

where

$$D_i = \sum_{j=i-n_i+1}^i (X_j - \mu_0) \tag{11}$$

$$n_i = \begin{cases} n_{i-1} + 1 & ,\text{if } C_i > 0 \\ 1 & ,\text{otherwise} \end{cases} \tag{12}$$

This research focused on individual observation, which is $n_i = 1$. It is suggested setting k equals to zero when employing the CUSUM in equation (13). The UCL of the ARL's value for three variables as follows.

Table 1. On Target ARL for MCUSUM Control Chart

UCL	ARL
5	131.47
5.5	207.56
6	323.03
6.5	493.15
7	766.07
7.5	1204.42

2.2 Methods

2.2.1 Data

The data source in this study is secondary data obtained from the PCC cement compressive strength test data for 3-days, 7-days, and 28-days. Retrieval of data in the period January to March 2018.

2.2.2 Research Variable

The research variables used in this study are the three main variables that influence the compressive strength of PCC type cement. The three variables used in this study are:

Table 2. Research Variables

Variables	Compressive Strength	Units	Specification Limits
X_1	3 days	Kg/cm ³	min 125
X_2	7 days	Kg/cm ³	min 200
X_3	28 days	Kg/cm ³	min 250

2.2.3 Data Structure

The data structure used in the PCC type cement compressive strength research is as follows:

Table 3. Data Structure

j - obser- vations	i - variables			
	x_1	x_2	..	x_p
1	x_{11}	x_{12}	..	x_{1p}
2	x_{21}	x_{22}	..	x_{2p}
3	x_{31}	x_{32}	..	x_{3p}
:	:	:	..	:
N	x_{n1}	x_{n2}	..	x_{np}

with

i : Many variables, where $i = 1, 2, \dots, p$; $p = 3$.

j : Many observations, where $j = 1, 2, \dots, n$; $n = 85$.

2.2.4 Research Steps

The steps in determining CUSUM control chart are as follows:

1. Conduct a literature test on the CUSUM control chart and its application.
2. Collecting secondary data about PCC cement compressive strength, which are compressive strength of 3-days, the compressive strength of 7-days and compressive strength of 28-days.
3. Perform a normal distribution test for each variable and conduct a correlation test between variables.
4. Apply the tabular CUSUM control chart on the PCC type cement compressive strength data.
5. Identifying uncontrolled data by determining process target values.
6. Perform analysis on the resulting CUSUM control chart.

The steps in determining MCUSUM control chart are as follows:

1. Calculate covariance matrices based on data.
2. Let be the standardized value of data.
3. Apply the MCUSUM control chart.
4. Identify uncontrolled data based on UCL values in the table.
5. Perform analysis on the resulting MCUSUM control chart.

3. RESULT AND DISCUSSION

To find a general description of each quality characteristic for the period of January - March 2018 on the compressive strength

of PCC type cement as follows.

Table 4. Description of Characteristics of Secondary Data of The PCC Cement Compressive Strength

Variables	Mean	Varian	Min	Max
X ₁	169	121,958	141	198
X ₂	235	145,282	200	267
X ₃	324	187,026	281	378

Based on Table 4 the minimum value of each variable does not exceed the specification limits set by the National Standardization Agency. Then, it can be seen that the value of variance is valued at the compressive strength variable of 3 days, which is 12.1968. This shows that the compressive strength data of 3 days from January to March 2018 requires homogeneity because the distance between data is relatively small.

3.1 Normal Distribution Test

A normal distribution test is done to find out how many per-versions on the variables that are owned. The following table is the result of the normal distribution test of each variable by selecting.

Table 5. Normal Distribution Test for PCC Compressive Strength of Cement

Variables	P-Value
3-Day Compressive Strength (X ₁)	0,911
7-Day Compressive Strength (X ₂)	0,607
28-Day Compressive Strength (X ₃)	0,242

Based on Table 5 it is known that each variable has a P-value that is greater than $\alpha=0.05$. So it is obtained that each variable is normally distributed.

3.2 Correlation Test

A correlation test is performed to determine the linear relationship between variables. The following table shows the results of correlation tests between variables.

Table 6. Correlation Test for PCC Compressive Strength of Cement

Variable	Pearson Correlation	
	X ₁	X ₂
X ₂	0,917	
X ₃	0,845	0,920

Based on Table 6 it is known that each variable has a value of rxy close to 1. This means that each variable has a strong correlation with other variables.

3.3 Application of CUSUM Control Chart

The initial step taken in applying the CUSUM control chart is to determine the UCL and LCL of each variable based on the average and variance that has been obtained previously. Select the value $h = \pm 5$ to determine UCL and LCL. Following is a table that contains UCL and LCL in applying the CUSUM control chart to the compressive strength of PCC type cement.

Table 7. UCL and LCL Values on PCC Type Cement Compressive Strength

Variables	UCL	LCL
X1	6,097,899	-6,097,899
X2	7,264,098	-7,264,098
X3	9,351,299	-9,351,299

After obtaining UCL and LCL from all variables, positive and negative tubular CUSUM values will be determined, and sequential periods of tubular CUSUM for each variable. The following will be presented a table of CUSUM tubular and sequential periods of tubular CUSUM for 3-days compressive strength.

Table 8. Tubular CUSUM Value and Sequential Period of Tubular CUSUM for 3-Days Compressive Strength

No	X _i	C _i ⁺	C _i ⁻	N ⁺	N ⁻
1	176	0,927	0,000	1	0
2	178	3,854	0,000	2	0
3	184	12,889	0,000	3	0
⋮	⋮	⋮	⋮	⋮	⋮
85	176	0,639	0,000	1	0

Based on Table 8. the following CUSUM control chart is obtained.

Based on Figure 1. it is known that there are uncontrolled data, namely the observations 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 30, 31, 32, 33, 34, 35, 36, 60, 61, 62, 63, 65 and 66.

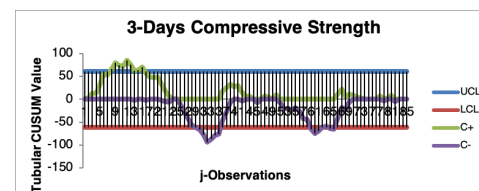


Figure 1. CUSUM Control Chart 3-Days Compressive Strength

The following will be presented a table of CUSUM tubular and sequential periods of tubular CUSUM for 7-days compressive strength. Based on Table 9. the following CUSUM control chart is obtained.

Based on Figure 2. it is known that there are uncontrolled data, namely the 8, 9, 10, 11, 12, 13, 33, 61 and 62 observations.

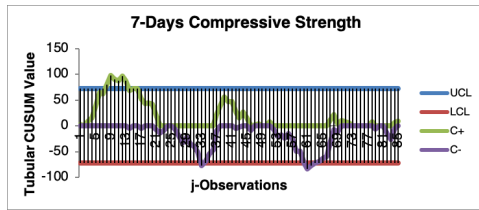


Figure 2. CUSUM Control Chart 7-Days Compressive Strength

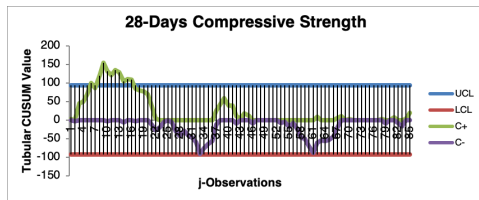


Figure 3. CUSUM Control Chart 28-Day Compressive Strength

The following will be presented a table of CUSUM tubular and sequential periods of tubular CUSUM for 28-days compressive strength. Based on Table 10. the following CUSUM control chart is obtained.

Based on Figure 3 it is known that there are uncontrolled data that is on observations 6, 8, 9, 10, 11, 12, 13, 14, 15 and 16.

Based on Figure 1. Figure 2. and Figure 3 it can be seen that for 85 samples of compressive strength data of PCC type cement in 3 days, compressive strength has a UCL of 60.99799 and an LCL of -60.97899 there are 23 out of control data. In 7 days, compressive strength has a UCL of 72.64098 and an LCL of -72.64098. There are nine data out of control. In 28 days compressive strength has a UCL of 93.51299 and an LCL of -93.51299, there are 10 data out of control. So it can be concluded that the mean target shift occurred. Here are the results of the calculation of the new target mean value (μ), to find out how big is the shift in the mean value of the target μ_0 .

From Table 11. it is clear that there is a shift in the value of each sample that is out of control, causing the CUSUM control chart to be out of control. After conducting the previous correlation test, it is known that for each compressive strength variable has a good correlation between one and the other. This is proven by the fact that for every sample that is not controlled for one variable, it will affect the state of the other variables in the next stage.

Table 9. Tubular CUSUM Value and Sequential Period of Tubular CUSUM for 7-Day Compressive Strength

No	X_i	C_i^+	C_i^-	N^+	N^-
1	245	2,882	0,000	1	0
2	234	0,000	0,000	0	0
3	251	8,925	0,000	1	0
⋮	⋮	⋮	⋮	⋮	⋮
85	244	8,629	0,000	1	0

Table 10. Tubular CUSUM Value and Sequential Period of Tubular CUSUM for 28-Day Compressive Strength

No	X_i	C_i^+	C_i^-	N^+	N^-
1	335	2,141	0,000	1	0
2	311	0,000	-3,156	0	1
3	378	45,196	0,000	1	0
⋮	⋮	⋮	⋮	⋮	⋮
85	349	19,282	0,000	1	0

3.4 Application of MCUSUM Control Chart

The multivariate CUSUM procedures are based on quadratic forms of the mean vector. Here, the difference between the two multivariate CUSUM procedures discussed centers on the point at which the accumulation is made. The following is the covariance matrix of the PCC type of compressive strength data.

$$\Sigma_0 = \begin{bmatrix} 148.6786 & 162.3398 & 192.6848 \\ 162.3398 & 210.9870 & 249.7453 \\ 192.6848 & 249.7453 & 394.6458 \end{bmatrix} \quad (13)$$

The result MCUSUM value based on the covariance matrix of the PCC type of the compressive strength data as follows.

Based on Table 1 about UCL values and Table 12 about MCUSUM value are obtained the MCUSUM control chart as follows.

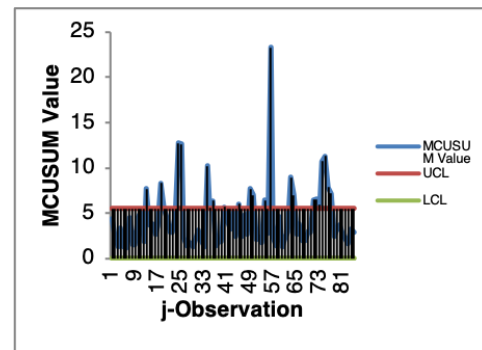


Figure 4. MCUSUM Control Chart for the ARL = 131.47

Based on figure 4 until figure 9, it is had that there are out of controls of MCUSUM value. It is known that the small ARL value had more out of control data than the big ARL value. It is shown that the smallest ARL value had the best specification control chart. The out of control data in MCUSUM control chart for the ARL = 1204.42 as follows.

The CUSUM and MCUSUM control charts are suitable for data that have a small mean shift. From the results of table 11 and table 13, it is obtained that the out of control data in the CUSUM control chart of each variable is not the same as the out of control data in the MCUSUM control chart. It is shown that uncontrolled observations on the cement compressive strength variables of 3-days, 7-days, and 28-days as measured univariately,

Table 11. The Result of Calculation of New Mean Target Value (μ) and Big Shift of Mean Target Value

Compressive Strength	Observation to	Mean new target (μ)	Shifting the Target Mean Value
3-Days	8	183.375	14.4
	9	183.889	14.914
	10	182.5	13.525
	11	181.636	12.661
	12	182.167	13.192
	13	180.923	11.948
	14	179.5	10.525
	15	179.333	10.358
	16	179.5	10.525
	17	178.405	9.43
	30	175.132	6.157
	31	173.988	5.013
	32	173.986	5.011
	33	174.546	5.571
	34	172.511	3.536
	35	170.74	1.765
	36	169.823	0.848
	60	170.095	1.12
	61	170.293	1.318
	62	169.218	0.243
63	168.028	-0.947	
65	167.394	-1.581	
66	167.235	-1.74	
7-Days	8	252.154	17.301
	9	253.026	18.172
	10	251.123	16.27
	11	250.112	15.259
	12	250.103	15.249
	13	248.864	14.011
	33	237.304	2.45
	61	235.979	1.125
62	234.633	-0.22	
28-Days	6	357.75	34.242
	8	352.833	29.326
	9	354.857	31.349
	10	349.375	25.867
	11	346.222	22.714
	12	346.3	22.792
	13	344.545	21.038
	14	341.5	17.992
	15	341.307	17.8
	16	340.571	17.064

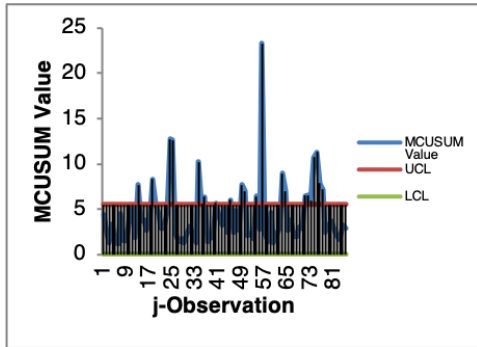


Figure 5. MCUSUM Control Chart for the ARL = 207.56

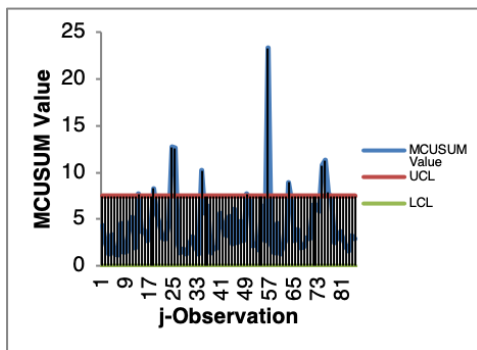


Figure 6. MCUSUM Control Chart for the ARL = 323.03

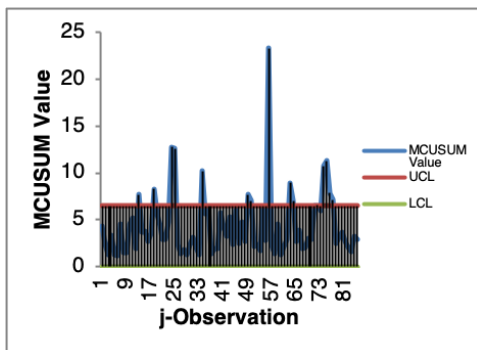


Figure 7. MCUSUM Control Chart for the ARL = 493.15

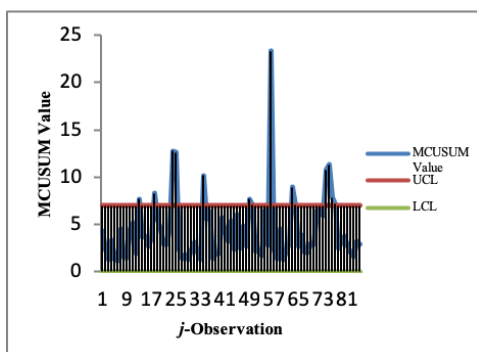


Figure 8. MCUSUM Control Chart for the ARL = 766.07

Table 12. MCUSUM Value of Three Variables

No	MCUSUM Value
1	4.353917868
2	2.382446771
3	1.302282022
⋮	⋮
85	2.873465687

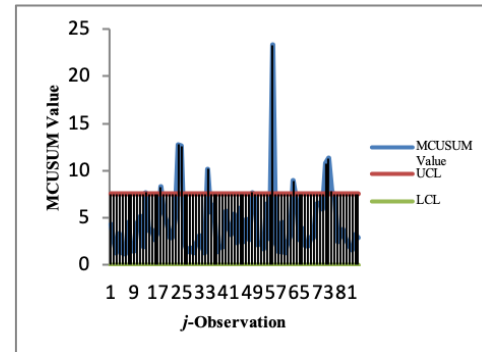


Figure 9. MCUSUM Control Chart for the ARL = 1204.42

it is obtained different out of control observations when measured with multivariate control charts. So for statistical quality control, the variables that have correlations are more suitable to use multivariate control charts.

4. CONCLUSIONS

CUSUM control chart for cement compressive strength test data in 3 days, 7 days and 28 days on PCC type cement produces an uncontrolled control chart. Because the compressive strength variables of 3 days, 7 days and 28 days have strong correlations, the out of control observations that occur in one variable also affect other variables. For moderately high correlated variables, processing with multivariate control charts will result in better control charts with less out of control observations compared to univariate control charts. In this study, the multivariate control chart used is the MCUSUM control chart, and the control chart used for univariate control charts is the CUSUM control chart. The two control charts provide different characteristics, but the MCUSUM control chart is more sensitive than the CUSUM control chart in controlling observations of data out of control.

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Table 13. The Out of Control Data of MCUSUM Control Char

No	Observation to	MCUSUM Value
1	13	7.704454037
2	18	8.295241371
3	24	12.74814526
4	25	12.62204634
5	34	10.19490286
6	49	7.7067801
7	56	23.3122278
8	63	8.939278926
9	74	10.7477131
10	75	11.30340619
11	76	7.841282376

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