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Research Paper

Set Covering Model in Solving Multiple Cutting Stock Problem

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Abstract
Cutting Stock Problem (CSP) is the determination of how to cut stocks into items with certain cutting rules. A diverse set of stocks is called multiple stocks. This study used the Pattern Generation (PG) algorithm to determine the cutting pattern of three sizes of stocks, then formulated them into the Gilmore and Gomory Model. The set covering model was generated from the Gilmore and Gomory model. There are two stages of cutting where the first stage is based on the length and the second stage is based on the width. Based on the results, selected cutting patterns in the first stage can be used in the second stage. The combination of patterns produced in the Gilmore and Gomory model showed that the use of stocks is less than the use of stocks in the set covering model.

Keywords
Cutting Stock Problem, Pattern Generation, Set Covering Model

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1. INTRODUCTION

Integer Linear Programming (ILP) is one of the optimization sciences applied in the paper industry. The problem with how to cut paper is known as the Cutting Stock Problem (CSP). CSP is the determination of how to cut stocks into items with certain cutting rules. Stock is the basic material used before formed into an item. Item is manufactured according to the size requested by customers. Diverse stock sizes are known as multiple stocks. Stocks that cut into the item, often result in remaining cuts called trim loss. Trim loss in large quantities will increase production costs. A good strategy to overcome this problem is by minimizing trim loss.

CSP was first examined by Kantorovich, followed by Gilmore and Gomory who were also successful in formulating CSP. CSP now has been developed by many researchers. Pattern Generation (PG) algorithm was proposed Suliman (2001) to determine feasible cutting patterns. The improved algorithm called Modified Branch and Bound Algorithm was also created (Rodrigo et al., 2012; Rodrigo, 2013, 2017). But the search for cutting patterns required a lot of time and a high level of accuracy so it took an application that can help to solve it. The application to form cutting patterns using Modified Branch and Bound Algorithm in two-dimensional CSP (Octarina et al., 2017, 2018, 2019) was improved so that the search for a lot number of paper cutting patterns could be done easily.

Resulting cutting patterns were formed into a model to obtain the optimal trim loss (Bangun et al., 2019, 2020; Octarina et al., 2020). Gilmore and Gomory proposed a model for two-dimensional CSP by extending the Column Generation Technique (CGT) approach in a one-dimensional CSP. CGT was a method that can provide the best solution in completing linear programs, because of the approach to solve large-scale linear program problems. Some of the researches about PG and CGT have been done ((Brandão et al., 2018; Mellouli and Dammak, 2008; Ma et al., 2018; Octarina et al., 2019).

Gilmore and Gomory model also can be developed into different formulations using the set covering model. The set covering model can solve problems involving multiple rows and columns. Umetani and Yagiura (2007) stated that set covering was one of the combinatorial optimization problems and can solve various difficult problems. Caprara et al. (2000) developed the heuristic method for completing the set covering model. They explained that there was no appropriate algorithm in the literature for the set covering model rather than using a good programming tool. Set covering research for various dimensions continues to develop. Jin et al. (2015) proposed a heuristic algorithm to find the initial solutions of CSP by considering defective and non-defective stocks. This algorithm was effective for two-dimensional stocks but could not eliminate the same patterns.

There have been limited studies concerned on the model of multiple CSP. Therefore, this research intends to implement the set covering model in the multiple two-dimensional CSP. The set covering model was completed by the LINGO 13.0 program.

2. EXPERIMENTAL SECTION

2.1 Data

The data used in this study consisted of 3 types of stock sizes and four different item sizes. The stock sizes are 1022×1200 mm^2,1200×1200 mm^2, and 1200×1500 mm^2 respectively with the item sizes are 282×208 mm^2,280×250 mm^2,235×185 mm^2, and 164×100 mm^2 of 20 pieces each. For detail, it can be seen in Table 1.

Table 1. Size of items and demands

No	Size of items (mm^2)	Demands (pieces)
1	282 x 208	20
2	280 x 250	20
3	235 x 185	20
4	164 x 100	20

2.2 Methods

The steps taken in this study are as follows:

a. Describe the data needed in forming cutting patterns which include stock size (length and width) and the number of demand for each stock.

b. Process data using the PG algorithm to determine cutting patterns with minimum trim loss.

c. Create a tree diagram that has been completed with the PG algorithm.

d. Form tables of cutting patterns are made by the sequence of branches so that cut loss can be obtained.

e. Formulate the Gilmore and Gomory model whereas the objective function shows the minimum amount of stock to meet the demand for each item and the constraints ensure that the strips produced in the first stage are those used in the second stage.

f. Solve the Gilmore and Gomory model using the LINGO 13.0 application.

g. Formulate the set covering model in the following ways:

1. Define the variables.

2. Determine the objective function that produces the minimum amount of stock to fulfill the demand for each item.

3. Determine constraints by ensuring that all requests are fulfilled.

h. Solve Set Covering model using the LINGO 13.0 application.

i. Analyze the final results.

3. RESULTS AND DISCUSSION

3.1 Pattern Generation

Pattern Generation (PG) is one algorithm to determine cutting patterns. Cutting pattern obtained using PG in CSP is usually obtained from the size of stocks with standard width w_k *(k=1,2,...,h)* cut into items with width w_i and length l_i*(i=1,2,...,n)*. The CSP model is as follow: Minimize

$$z = \sum_{k=1}^{h}\sum_{j=1}^{m} c_{jk}x_{jk} + \sum_{i=1}^{n} W_i S_i$$

Subject to

$$\sum_{k=1}^{h}\sum_{j=1}^{m} a_{jk}x_{jk} - S_i = I_i \tag{1}$$

$$x_{kj}, S_i, c_{jk}, a_{ijk} \geq 0, \text{for all } i, j \text{ and } k$$

whereas

a_{ijk} = is the number of the item with the width w_i which will be cut according to j^{th} pattern from k^{th} pieces ($i = 1, 2, \ldots, n; j = 1, 2, \ldots, m; k = 1, 2, \ldots, h$)

x_{jk} = is the length of k^{th} pieces which will be cut according to j^{th} pattern

c_{jk} = is the trim loss

S_i = is the residual length which will produce item with the width W_i

m = is the number of cutting patterns

The PG algorithm is as follows:

1. Sort the length $l_(i), (i = 1, 2, \ldots, n)$ with descending order.

2. Fill the first column ($j_0 = 1$) of the matrix using Equation (2).

$$a_{ijk} = \left[\frac{(I'_k - \sum_{z=1}^{i-1} a_{zjk} I_z)}{w_i} \right] \tag{2}$$

3. Use Equation (3) to find the cut loss from the cutting pattern

$$c_{jk} = I'_k - \sum_{z=1}^{i-1} a_{zjk} I_z \tag{3}$$

4. Set index level (row index), i to $n - 1$.

5. Check current vertices at the i^{th} level, for initial vertex (i, j). If vertices have the value equals to zero ($a_{ijk} = 0$), go to Step 7. If not, generate the new column $j_p = j_{(p-1)}+1$ with these elements :

a. $a_{zik} = a_{(z(i-1)k,}(z = 1, \ldots, i - 1)$ element to fill preceding vertices i, j.

b. $a_{ijk} = a_{i(j-1)k} - 1$ element to fill current vertices i, j.

6. Fill remaining vertices from the j^{th} column, like $a_{(i+1)jk}, a_{(i+2)jk}, \ldots, a_{njk}$ using Equation (3).

7. Find the cut loss from the j^{th} cutting pattern using Equation (3). Go back to Step 4.

8. Set $i_p = i_{p-1} - 1$. If $i_p > 0$ redo Step 5. If not, stop.

The PG algorithm is used to determine the cutting pattern based on the length of two available stock sizes, namely 1022 mm and 1200 mm. By using the data in Table 1, the determination of the pattern based on length is described by the available length, and below is the detailed for the first pattern.

1. Sort the length of stocks in descending order, $l_k^{'}$ for $k = 1, 2$ so $l_1^{'}$=1200 mm and $l_2^{'}$=1022 mm.
2. Sort the length of items in descending order, l_i for $i = 1, 2, 3, 4$ so l_1=282 mm, l_2=280 mm, l_3=235 mm, and l_4=164 mm.
3. Fill the element of first row ($i = 1$) , first column ($j_0 = 1$) for $k = 1$ and $i = 1, 2, 3, 4$, by using Equation (2) :

$$a_{i11} = \left[\frac{(I_1^{'} - \sum_{z=1}^{i-1} a_{z11} I_z)}{I_i}\right]$$

$$a_{111} = \left[\frac{1200 - 0}{282}\right] = 0$$

$$a_{111} = \left[\frac{1200 - (4)(282)}{280}\right] = \left[\frac{72}{280}\right] = 0$$

$$a_{111} = \left[\frac{1200 - (4)(282) - (0)(280)}{235}\right] = \left[\frac{72}{235}\right] = 0$$

$$a_{111} = \left[\frac{1200 - (4)(282) - (0)(280) - (0)(235)}{164}\right] = \left[\frac{72}{164}\right] = 0$$

4. Find the cut loss from the first pattern by using Equation (3).

$$c_{11} = I_1^{'} - \sum_{z=1}^{i-1} a_{z11} = 1200 - (4)(282) - (0)(280) - (0)(235) - (0)(164) = 72$$

5. Set index level (row index) $i_0 = n - 1 = 4 - 1 = 3$
6. Find the new vertex from the third level $a_{311} = 0$.
7. Subtract 1 from index i_0 in Step 5, so $i_1 = 3 - 1 = 2$, $i_1 > 0$.
8. The current vertex is in the second level, $a_{211} = 0$.
9. Subtract 1 from index i_1 in Step 7, so $i_2 = 2 - 1 = 1$, $i_2 > 0$.
10. The current vertex is in the first level a_{111}, $a_{111} > 0$, so continue to the next pattern.

There are 11 cutting patterns according to the length of 1022 m and 14 cutting patterns according to the length of 1200 mm, which can be seen in Table 2 and Table 3 respectively.

From Table 2 for the stock with a length of 1022 mm, it can be seen that by using the first pattern, it will yield 3 pieces of items with length 282 mm and an item with a length 164 cm with 12 mm of cut loss, and so on.

From Table 3 for the stock with the length of 1200 mm, it can be seen that by using the first pattern, it will yield 3 pieces of items with length 282 mm and 2 pieces of items 164 mm with 26 mm of cut loss, and so on for the next pattern. According to the width, there are 38 cutting patterns of width 1200 mm and 65 cutting patterns of width 1500 mm, which can be seen in Table 4 and Table 5.

It can be seen from Table 4, there are 4 pieces of items of width 250 mm and an item of width 185 mm for the first cutting pattern. The trim loss for this first pattern is 15 mm. It continues until the 38^{th} cutting pattern. Meanwhile, from Table 5, we can see that the first pattern only yields 6 pieces of items of width 250 mm without cut loss. All the cutting patterns in Table 2-5 have a maximum of 60 mm of cut loss.

3.2 The Gilmore and Gomory Model

The Gilmore and Gomory model are as follows.

Minimize

$$z = \sum_{j\epsilon j_0}^{j} \lambda_j^0 \tag{4}$$

Subject to :

$$M^{'} \bar{\lambda} = 0$$

$$M^{''} \bar{\lambda} \geq b$$

$$\bar{\lambda} \geq 0 \text{and integer}$$

with

$M^{'}$ and $M^{''}$ are m' in the first row and m" in the last row of M.

$$\bar{\lambda} = [\lambda_1^0 ... \lambda_j^0 \lambda_1^1 ... \lambda_j^1 \lambda_1^2 ... \lambda_j^2 ... \lambda_1^{m^{'}} ... \lambda_j^{m^{'}}]^T$$

λ_j^0 is the j^{th} cutting pattern in the first stage.

λ_j^s is the j^{th} cutting pattern that related to sth pattern in the second stage, where $s\epsilon(1, \ldots, m^{'})$.

$b = [b_1 b_2 ... b_m]^T$ is the number of demand of item i.

The objective function in this case is to minimize the use of stocks. Meanwhile, the problem is how to determine the optimal cutting pattern with minimum stock usage. Cutting is done in the first stage along the length and then in the second stage along the width.

The Gilmore and Gomory model for the stock of 1022×1200 mm^2 can be seen in Equation (5) and Constraints (6)-(14).

Minimize

$$z = \sum_{j=1}^{11} \lambda_j^0 \tag{5}$$

Subject to

$$\sum_{j=0}^{4} \lambda_j^0 + \lambda_7^0 + 3(\lambda_6^0 + \lambda_9^0 + \lambda_{10}^0 + 2\lambda_{11}^0) - \lambda_1^1 = 0 \tag{6}$$

$$\lambda_3^0 + \lambda_6^0 + \lambda_9^0 + 3(\lambda_5^0 + \lambda_8^0) + 2\lambda_{10}^0 - \sum_{j=1}^{4} \lambda_j^2 = 0 \tag{7}$$

Table 2. Cutting patterns according to the length of 1022 mm

The *j*-th cutting pattern	The length for each cutting pattern 282 mm	280 mm	235 mm	164 mm	Cut Loss (mm)
1	3	0	0	1	12
2	2	1	0	1	14
3	2	0	1	1	59
4	1	2	0	1	16
5	1	0	3	0	35
6	1	0	1	3	13
7	0	3	0	1	18
8	0	1	3	0	37
9	0	1	1	3	15
10	0	0	2	3	60
11	0	0	0	6	38

Table 3. Cutting patterns according to the length of 1200 mm

The *j*-th cutting pattern	The length for each cutting pattern 282 mm	280 mm	235 mm	164 mm	Cut Loss (mm)
1	3	0	0	2	26
2	2	1	0	2	28
3	2	0	2	1	2
4	1	2	0	2	30
5	1	1	2	1	4
6	1	0	3	1	49
7	1	0	1	4	27
8	0	3	0	2	32
9	0	2	2	1	6
10	0	1	3	1	51
11	0	1	1	4	29
12	0	0	5	0	25
13	0	0	3	3	3
14	0	0	0	7	52

$$\lambda_2^0 + 2\lambda_4^0 + 3\lambda_7^0 + \lambda_8^0 + \lambda_9^0 - \sum_{j=1}^{10} \lambda_j^3 = 0 \tag{8}$$

$$3\lambda_1^0 + 2(\lambda_2^0 + \lambda_3^0) + \lambda_4^0 + \lambda_5^0 + \lambda_6^0 - \sum_{j=1}^{23} \lambda_j^3 = 0 \tag{9}$$

$$\begin{aligned} 12\lambda_1^1 + 10\lambda_2^1 + 8\lambda_2^2 + 6\lambda_3^2 + 4\lambda_4^2 + 9\lambda_1^3 + 2\lambda_2^3 \\ +7\lambda_4^3 + 5\lambda_5^3 + 3\lambda_6^3 + \lambda_7^3 + 4\lambda_8^3 + 2\lambda_9^3 + 8\lambda_1^4 + 6\lambda_2^4 \\ +4\lambda_3^4 + 2\lambda_4^4 + 4\lambda_5^2 + 4\lambda_4^5 + 2\lambda_6^4 + 2\lambda_8^4 + \lambda_{10}^4 + 7\lambda_{11}^4 + 5\lambda_{12}^4 \\ +5\lambda_{14}^4 + 3\lambda_{15}^4 + 3\lambda_{15}^4 + 3\lambda_{16}^4 + \lambda_{17}^4 + \lambda_{18}^4 + 3\lambda_{19}^4 + \lambda_{20}^4 + 2\lambda_{21}^4 \geq 20 \end{aligned} \tag{10}$$

$$\begin{aligned} \lambda_1^2 + 2\lambda_2^2 + 3\lambda_3^2 + 4\lambda_4^2 + 4\lambda_3^2 + 5\lambda_3^3 + \lambda_5^3 + 2\lambda_6^3 + \\ 3\lambda_7^3 + \lambda_3^{10} + \lambda_1^4 2\lambda_4^2 + 3\lambda_3^4 + 4\lambda_4^4 + 2\lambda_5^4 + 3\lambda_6^4 + \\ 4\lambda_7^4 + 2\lambda_8^4 + 3\lambda_9^4 + 4\lambda_{13}^4 + \lambda_{15}^4 + \lambda_{17}^4 + 2\lambda_{20}^4 + \lambda_{22}^4 \geq 20 \end{aligned} \tag{11}$$

$$\begin{aligned} \lambda_1^3 + \lambda_2^{32} + 2\lambda_5^3 + 2\lambda_6^3 + 2\lambda_7^3 + 3\lambda_8^3 + 4\lambda_9^3 + 4\lambda_{10}^3 + 4\lambda_{11}^4 + \lambda_{11}^4 + \lambda_{13}^4 + \\ \lambda_{14}^4 + \lambda_{15}^4 + \lambda_{16}^4 + \lambda_{17}^4 + \lambda_{18}^4 + 2\lambda_{19}^4 + 2\lambda_{20}^4 + 3\lambda_{21}^4 + 3\lambda_{22}^4 + 3\lambda_{23}^4 \geq 20 \end{aligned} \tag{12}$$

$$\begin{aligned} \lambda_1^4 + \lambda_2^4 + \lambda_3^4 + \lambda_4^4 + 2\lambda_5^4 + 2\lambda_6^4 + 2\lambda_7^4 + 3\lambda_8^4 + 3\lambda_9^4 + 5\lambda_{10}^4 \\ +\lambda_{11}^4 + \lambda_{12}^4\lambda_{13}^4 + 2\lambda_{14}^4 + 2\lambda_{15}^4 + 3\lambda_{16}^4 + 3\lambda_{17}^3 + 4\lambda_{18}^4 + \\ \lambda_9^3 + 4\lambda_{10}^3 + 4\lambda_{11}^4 + \lambda_{19}^4 + \lambda_{20}^4 + \lambda_{21}^4 + \lambda_{22}^4 + \lambda_{23}^4 \geq 20 \end{aligned} \tag{13}$$

$$\lambda \geq 0 \tag{14}$$

Table 4. Cutting patterns according to the width of 1200 mm

The j-th cutting pattern	The width for each cutting pattern (mm)				Cut Loss (mm)	The j-th cutting pattern	The width for each cutting pattern (mm)				Cut Loss (mm)
	250	208	185	100			250	208	185	100	
1	4	0	1	0	15	20	1	1	0	7	42
2	4	0	0	2	0	21	1	0	5	0	25
3	3	2	0	0	34	22	1	0	4	2	10
4	3	1	1	0	57	23	1	0	0	9	50
5	3	1	0	2	42	24	0	5	0	1	60
6	3	0	0	4	50	25	0	3	3	0	21
7	2	1	2	1	22	26	0	3	2	2	6
8	2	1	1	3	7	27	0	2	4	0	44
9	2	0	3	1	45	28	0	2	3	2	29
10	2	0	2	3	30	29	0	2	2	4	14
11	2	0	1	5	15	30	0	1	4	2	52
12	2	0	0	7	0	31	0	1	3	4	37
13	1	4	0	1	18	32	0	1	2	6	22
14	1	3	1	1	41	33	0	1	1	8	7
15	1	3	0	3	26	34	0	0	4	4	60
16	1	2	1	3	49	35	0	0	3	6	45
17	1	2	0	5	34	36	0	0	2	8	30
18	1	1	4	0	2	37	0	0	1	10	15
19	1	1	1	5	57	38	0	0	0	12	0

with

$$\lambda = [\lambda_1^0 ... \lambda_{11}^0 \lambda_1^1 \lambda_1^2 ... \lambda_4^2 \lambda_1^3 ... \lambda_{10}^3 ... \lambda_{23}^4]^T$$

Constraints (6-9) show that strips with lengths of 164 mm, 235 mm, 280 mm, and 282 mm are used in the first and second stages of the cutting pattern. Constraints (10-13) show that items measuring 164×100 mm^2, 235×185 mm^2, 280×250 mm^2, and 282×208 mm^2 are produced not less than 20 sheets. Constraint (14) shows the nonnegative solution. By using LINDO 13.0, the solution of model with Objective Function (5) and Constraints (6-14) shows that in the first stage $\lambda_1^0 = 3$ and $\lambda_7^0 = 1$ which means the $1^{st}$and the 7^{th} pattern will be used 3 times and once respectively. On the other side, in the second stage, the solution shows that $\lambda_9^4 = 6$ which means the 9th pattern on the fourth stripe will be used six times. The objective function $z = 4$ means that there are 4 pieces of the first stock of sizes 1022×1200 mm^2.

The Gilmore and Gomory model for the stock 1200×1200 mm^2 can be seen in Equation (15) and Constraints (10-13) to Constraints (16-20).

Minimize

$$z = \sum_{j=1}^{14} \lambda_j^0 \tag{15}$$

Subject to

$$2(\lambda_1^0 + \lambda_2^0 + \lambda_4^0 + \lambda_5^0 + \lambda_6^0) + \lambda_9^0 + \lambda_{10}^0 + 3\lambda_{13}^0 + 7\lambda_{14}^0 - \lambda_1^1 = 0 \tag{16}$$

$$2\lambda_3^0 + 2\lambda_5^0 + 3\lambda_6^0 + \lambda_7^0 + 2\lambda_9^0 + 3\lambda_{10}^0 + \lambda_{11}^0 + 5\lambda_{12}^0 + 3\lambda_{13}^0 - \sum_{j=1}^{4} \lambda_j^2 = 0 \tag{17}$$

$$\lambda_2^0 + 2\lambda_4^0 + \lambda_5^0 + 3\lambda_8^0 + 2\lambda_9^0 + \lambda_{10}^0 + \lambda_{11}^0 - \sum_{j=1}^{10} \lambda_j^3 = 0 \tag{18}$$

$$3\lambda_1^0 + 2\lambda_2^0 + 2\lambda_3^0 + \lambda_4^0 + \lambda_5^0 + \lambda_6^0 + \lambda_7^0 - \sum_{j=1}^{23} \lambda_j^4 = 0 \tag{19}$$

With Equations (10)-(13)

$$\lambda \geq 0 \tag{20}$$

with

$$\lambda = [\lambda_1^0 ... \lambda_{14}^0 \lambda_1^1 \lambda_1^2 ... \lambda_4^2 \lambda_1^3 ... \lambda_{10}^3 \lambda_1^4 ... \lambda_{23}^4]^T$$

By using LINDO 13.0, the solution of model with Objective Function (15) and Constraints (16-20) shows that the 1^{st} and the 3^{rd}pattern will be used once and three times respectively in the first stage. While in the second stage, we will use four times the 4^{th} pattern on the second stripe and once of the 18^{th} or the 23^{rd} pattern on the fourth stripe.

The Gilmore and Gomory model for the stock 1200×1500 mm^2 can be seen in Equation (21) and Constraints (22-30).

Table 5. Cutting patterns according to the width of 1500 mm

The j-th cutting pattern	The width for each cutting pattern (mm)				Cut Loss (mm)	The j-th cutting pattern	The width for each cutting pattern (mm)				Cut Loss (mm)
	250	208	185	100			250	208	185	100	
1	6	0	0	0	0	34	1	2	1	6	49
2	5	1	0	0	42	35	1	2	0	8	34
3	5	0	0	2	50	36	1	1	5	1	17
4	4	1	1	1	7	37	1	1	4	3	2
5	4	0	2	1	30	38	1	1	1	8	57
6	4	0	1	3	15	39	1	1	0	10	42
7	4	0	0	5	0	40	1	0	6	1	40
8	3	3	0	1	26	41	1	0	5	3	25
9	3	2	1	1	49	42	1	0	4	5	10
10	3	2	0	3	34	43	1	0	0	12	50
11	3	1	1	3	57	44	0	7	0	0	44
12	3	1	0	5	42	45	0	6	0	2	52
13	3	0	4	0	10	46	0	5	0	4	60
14	3	0	0	7	50	47	0	4	3	1	13
15	2	2	3	0	29	48	0	3	4	1	36
16	2	2	2	2	14	49	0	3	3	3	21
17	2	1	4	0	52	50	0	3	2	5	6
18	2	1	3	2	37	51	0	2	5	1	59
19	2	1	2	4	22	52	0	2	4	3	44
20	2	1	1	6	7	53	0	2	3	5	29
21	2	0	4	2	60	54	0	2	2	7	14
22	2	0	3	4	45	55	0	1	4	5	52
23	2	0	2	6	30	56	0	1	3	7	37
24	2	0	1	8	15	57	0	1	2	9	22
25	2	0	0	10	0	58	0	1	1	11	7
26	1	5	1	0	25	59	0	0	8	0	20
27	1	5	0	2	10	60	0	0	7	2	5
28	1	4	2	0	48	61	0	0	4	7	60
29	1	4	1	2	33	62	0	0	3	9	45
30	1	4	0	4	18	63	0	0	2	11	30
31	1	3	2	2	56	64	0	0	1	13	15
32	1	3	1	4	41	65	0	0	0	15	0
33	1	3	0	6	26						

Table 6. Optimal Solution

Type of Stock	Stage of Cutting	The Gilmore and Gomory Model		The Set Covering Model	
		Type of Pattern	Number of Cutting	Type of Pattern	Number of Cutting
Stock 1	First Stage	The 1st	3	The 2nd	12
		The 7th	1	The 4th	3
				The 8th	6
				The 9th	2
	Second Stage	Stripe 4		Stripe 3	
		The 9th	6	The 3rd	1
				The 9th	4
				The 10th	1
				Stripe 4	
				The 9th	6
				The 18th	1
Stock 2	First Stage	The 1st	1	The 1st	5
		The 3rd	3	The 2nd	1
				The 5th	13
				The 8th	2
	Second Stage	Stripe 4		Stripe 2	
		The 18th	1	The 4th	5
		The 23th	1	Stripe 4	
				The 10th	2
				The 23th	7
Stock 3	First Stage	The 1st	1	The 1st	4
		The 5th	1	The 5th	12
		The 8th	1	The 8th	4
	Second Stage	Stripe 3		Stripe 1	
		The 16th	1	The 1st	2
		Stripe 5		Stripe 2	
		The 15th	6	The 3rd	3
				Stripe 3	
				The 16th	4
				Stripe 4	
				The 15th	3

Minimize

$$z = \sum_{j=1}^{14} \lambda_j^0 \tag{21}$$

Subject to

$$2(\lambda_1^0+\lambda_2^0+\lambda_4^0+2\lambda_7^0+\lambda_8^0+2\lambda_{11}^0)+\lambda_3^0+\lambda_5^0+\lambda_6^0+\lambda_9^0+\lambda_{10}^0+3\lambda_{13}^0+7\lambda_{14}^0-\lambda_1^1 = 0 \tag{22}$$

$$2(\lambda_3^0+\lambda_5^0+\lambda_9^0)+3(\lambda_6^0+\lambda_{10}^0+\lambda_{13}^0)+\lambda_7^0+\lambda_{11}^0+5\lambda_{12}^0-\sum_{j=1}^{6}\lambda_j^2 = 0 \tag{23}$$

$$\lambda_2^0 + 2\lambda_5^0 + \lambda_5^0 + 3\lambda_8^0 + 2\lambda_9^0 + \lambda_{10}^0 + \lambda_{11}^0 - \sum_{j=1}^{15}\lambda_j^3 = 0 \tag{24}$$

$$3\lambda_1^0 + 2\lambda_2^0 + 2\lambda_3^0 + \sum_{j=45}^{7}\lambda_j^0 - \sum_{j=1}^{43}\lambda_j^4 = 0 \tag{25}$$

$$\begin{aligned} 15\lambda_1^1 + 13\lambda_1^2 + 11\lambda_2^2 + 9\lambda_3^2 + 7\lambda_4^2 + 2\lambda_5^2 + 12\lambda_1^3 + 5\lambda_2^3 + \\ 3\lambda_3^3 + \lambda_4^3 + 10\lambda_5^3 + 8\lambda_6^3 + 6\lambda_7^3 + 4\lambda_8^3 + 2\lambda_9^3 + 7\lambda_{10}^3 + 5\lambda_{12}^3 + 3\lambda_{13}^3 + \\ \lambda_{14}^3 + 2\lambda_{15}^3 + 11\lambda_1^4 + 9\lambda_2^4 + 7\lambda_3^4 + 5\lambda_4^4 + 7\lambda_5^4 + 5\lambda_6^4 + 3\lambda_7^4 + \lambda_8^{43} + 5\lambda_9^4 \\ +3\lambda_{10}^4 + \lambda_{11}^4 + \lambda_{12}^4 + 4\lambda_{13}^4 + 2\lambda_{14}^4 + 10l\lambda_{16}^4 + 8\lambda_{17}^4 + 3\lambda_{18}^4 + \\ \lambda_{19}^4 + 8\lambda_{10}^4 + 6\lambda_{21}^4 + 6\lambda_{22}^4 + 4\lambda_{23}^4 + 2\lambda_{24}^4 + \lambda_{39}^4 + \lambda_{40}^4 + \lambda_{41}^4 \geq 20 \end{aligned} \tag{26}$$

$$\begin{aligned}\lambda_1^2 + 2\lambda_{14}^3 + \lambda_1^4 + 2\lambda_2^4 + 7\lambda_5^2 + 8\lambda_6^2 + 4\lambda_2^3 + 5\lambda_3^3 + 6\lambda_4^3 + \lambda_6^3 + 2\lambda_7^3 + 3\lambda_8^3 + \\ 4\lambda_9^3 + 4\lambda_{11}^3 + \lambda_{13}^3 + 2\lambda_{14}^3 + \lambda_1^4 + 2\lambda_2^4 + 3\lambda_3^4 + 2\lambda_{15}^3 + 11\lambda_1^4 + \\ 9\lambda_2^4 + 4\lambda_4^4 + 2\lambda_5^4 + 3\lambda_6^4 + 7\lambda_6^4 + 5\lambda_8^4 + 2\lambda_9^{43} + 3\lambda_{10}^4 + 3\lambda_{10}^4 + \\ 4\lambda_{11}^4 + 3\lambda_{12}^4 + 4\lambda_2^4 + 5\lambda_3^4 + 6\lambda_4^4 + \lambda_6^4 + 4\lambda_7^4 + 5\lambda_8^4 + \lambda_{10}^4 + \\ 3\lambda_{12}^4 + \lambda_{17}^4 + 4\lambda_{18}^4 + 5\lambda_{19}^4 + \lambda_{21}^4 + \lambda_{23}^4 + 2\lambda_{24}^4 + \lambda_{30}^4 + 2\lambda_{31}^4 + \\ 3\lambda_{32}^4 + 4\lambda_{33}^4 + 2\lambda_{34}^4 + 3\lambda_{24}^4 + \lambda_{37}^4 + \lambda_{39}^4 + \lambda_{41}^4 \geq 20\end{aligned} \tag{27}$$

$$\begin{aligned}\sum_{j=1}^{4} \lambda_j^4 + 2\sum_{j=5}^{9} \lambda_j^3 + 3\sum_{j=10}^{11} \lambda_j^3 + 4\sum_{j=12}^{14} \lambda_j^3 + 5\lambda_{15}^3 + 6\lambda_{16}^3 + 7\lambda_{17}^3 + \\ \sum_{j=16}^{33} \lambda_j^4 + 2\sum_{j=30}^{35} \lambda_j^4 + 3\sum_{j=36}^{40} \lambda_j^4 + 4\lambda_{41}^4 + 5\lambda_{42}^4 \geq 20\end{aligned} \tag{28}$$

$$\begin{aligned}\sum_{j=1}^{4} \lambda_j^4 + 2\sum_{j=5}^{8} \lambda_j^4 + 3\sum_{j=9}^{11} \lambda_j^4 + 4\lambda_{12}^4 + 5\lambda_{13}^4 + 6\lambda_{14}^4 + 7\lambda_{14}^4 + \sum_{j=16}^{33} \lambda_j^4 + \\ \sum_{j=16}^{33} \lambda_j^4 + 2\lambda_{34}^4 + 2\lambda_{35}^4 + \lambda_{36}^4 + \lambda_{37}^4 + \lambda_{38}^4 + 2\lambda_{39}^4 + 3\lambda_{40}^4 + \lambda_{41}^4 + \lambda_{42}^4 + \geq 20\end{aligned} \tag{29}$$

$$\lambda \geq 0 \tag{30}$$

with

$$\lambda = [\lambda_1^0 ... \lambda_{14}^0 \lambda_1^1 \lambda_1^2 ... \lambda_6^2 \lambda_1^3 ... \lambda_{16}^3 \lambda_1^4 ... \lambda_{42}^4]^T$$

16^{th} pattern will be used once on the third stripe in the second stage and the 15th patterns on the fourth stripe will be used six times.

3.3 Set Covering Model

Set covering model can be seen in Equation (31).

Minimize

$$z = \sum_{j=1}^{n} x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_1, \text{i = 1,2, ..., m} \tag{31}$$

$$x_j \epsilon Z^+, \text{i = 1,2, ..., n}$$

whereas

x_j is the number of j^th cutting pattern

a_{ij} is the number of i^th items which cut in j^th cutting pattern

b_i is the number of demands

From all cutting patterns, then formulated the set covering model. The model for a stock size of 1022 mm × 1200 mm is:

Minimize

$$z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \tag{32}$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + 3x_6 + x_7 + 3x_9 + 3x_{10} + 6x_{11} - y_1 \geq 20 \tag{33}$$

$$x_3 + 3x_5 + x_6 + 3x_8 + x_9 + 2x_{10} - \sum_{i=2}^{5} y_i \geq 20 \tag{34}$$

$$x_2 + 2x_4 + 3x_7 + x_8 + x_9 - \sum_{i=6}^{15} y_i \geq 20 \tag{35}$$

$$3x_1 + 2x_2 + 2x_3 + x_4 + x_5 + x_6 - \sum_{i=16}^{38} y_i \geq 20 \tag{36}$$

$$\begin{aligned}12_1 + 10y_2 + 8y_3 + 6y_4 + 4y_5 + 8y_6 + 6y_7 + 4y_8 + 2y_9 + 4y_{10} + \\ 2y_{11} + 2y_{13} + y_{15} + 9y_{16} + 2y_{17} + 7y_{19} + 5y_{20} + 5y_{22} + 3y_{23} + \\ 3y_{24} + y_{25} + y_{26} + 7y_{27} + 5y_{28} + 3y_{29} + y_{30} + 3y_{31} + y_{32} + 4y_{33} + \\ 2y_{34} + 2y_{37} \geq 20\end{aligned} \tag{37}$$

$$\begin{aligned}y_2 + 2y_3 + 3y_4 + 4y_5 + y_6 + 2y_7 + 3y_8 + 4y_9 + 2y_{10} + 3y_{11} \\ +4y_{12} + 2y_{13} + 3y_{14} + 4y_{17} + 5y_{18} + y_{20} + 4y_{21} + y_{23} + \\ y_{25} + y_{28} + 2y_{29} + 3y_{30} + y_{31} + 2y_{32} + y_{35} + y_{38} \geq 20\end{aligned} \tag{38}$$

$$\begin{aligned}y_6 + y_7 + y_8 + 2y_9 + 2y_{10} + 2y_{11} + 2y_{12} + 3y_{13} + 4y_{14} \\ +4y_{15} + y_{26} + y_{27} + y_{28} + y_{29} + y_{30} + y_{31} + y_{32} + y_{33} + \\ 2y_{34} + 2y_{35} + 3y_{36} + 3y_{37} + 3y_{38} \geq 20\end{aligned} \tag{39}$$

$$\begin{aligned}y_{16} + y_{17} + y_{18} + y_{19} + 2y_{20} + 2y_{21} + 2y_{22} + 3y_{23} \\ +3y_{24} + 5y_{25} + y_{26} + y_{27} + y_{28} + 2y_{29} + 2y_{30} + 3y_{31} \\ +3y_{32} + 4y_{33} + y_{34} + y_35 + y_{36} + y_{37} + 2y_{38} \geq 20\end{aligned} \tag{40}$$

$x_j, y_j \geq 0$ and integer, $j = 1, 2, \ldots, 38$.

By using the LINGO 13.0, the optimal solutions of model with Objective Function (32) and Constraints (33-40) are $x_2 = 12, x_4 = 3, x_9 = 2, x_8 = y_{24} = 6, y_1 = y_8 = y_{15} = y_{33} = 1, y_{14} = 4$ with the objective value $z = 23$ Based on the solutions, from the first stage of cutting (cutting based on length), the 2^{nd} cutting pattern was cut 12 times, the 4^{th} cutting pattern was cut 3 times, the 8^{th} and the 9^{th} cutting pattern were cut 6 times and twice respectively. In the second stage of cutting, the first cutting pattern was cut once in the first stripe. None cutting pattern was chosen in the second stripe. In the third stripe, the 3^{rd} and 10^{th} were used once each and the 9^{th} cutting pattern was used four times. Only the 18^{th} cutting pattern was used once in the fourth stripe.

Set covering model for stock size of 1200 mm × 1200 mm is:

Minimize

$$z = x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}+x_{11}+x_{12}+x_{13}+x_{14} \quad (41)$$

$$2x_1+2x_2+x_3+2x_4+x_5+x_6+4x_7+2x_8+x_9+x_{10}+4x_{11}+3x_{13}+7x_{14}-y_1 \geq 20 \quad (42)$$

$$2x_3+2x_5+3x_6+x_7+2x_9+3x_{10}+x_{11}+5x_{12}+3x_{13}-\sum_{i=2}^{5} y_i \geq 20 \quad (43)$$

$$x_2+2x_4+x_5+3x_8+2x_9+x_{10}+x_{11}-\sum_{i=6}^{5} y_i \geq 20 \quad (44)$$

$$3x_1+2x_2+2x_3+x_4+x_5+x_6+x_7-\sum_{i=6}^{38} y_i \geq 20 \quad (45)$$

With Equations (37-40)

$x_j, y_j \geq 0$, and integer $j = 1, 2, \ldots, 38$.

The optimal solutions of model with Objective Function (41) and Constraints (37-40) to Contraints (42-45) are $x_1 = 5, x_2 = 1, x_5 = 13, x_8 = 2, y_5 = 5, y_{25} = 2, y_{38} = 7$ with objective value z=21. The first cutting pattern (cut 5 times), the second cutting pattern (cut once), the 5^{th} cutting pattern (cut 13 times), and the 8^{th} cutting pattern (cut twice) were used in the first stage. For the second stage, it used the second and fourth stripe. The 4^{th} cutting pattern was used 5 times in the second stripe. The 10^{th} cutting pattern was used twice and the 23^{rd} cutting pattern was used 7 times each in the fourth stripe.

Set Covering model for stock size of 1200 mm × 1500 mm is Minimize (41) Subject to Constraints (42)

$$2x_3+2x_5+3x_6+x_7+2x_9+3x_{10}+x_{11}+5x_{12}+3x_{13}-\sum_{i=2}^{7} y_i \geq 20 \quad (46)$$

$$x_2+2x_4+x_5+3x_8+2x_9+x_{10}+x_{11}-\sum_{i=8}^{65} y_i \geq 20 \quad (47)$$

$$3x_1+2x_2+2x_3+x_4+x_5+x_6+x_7-\sum_{i=24}^{65} y_i \geq 20 \quad (48)$$

With Equations (37-40)

$x_j, y_j \geq 0$ and integer $j = 1, 2, \ldots, 65$.

The optimal solution of Model with Objective Function (41) and Constraints (37-40) to Contraints (42), (46-48) are $x_1 = 4, x_5 = 12, x_8 = 4, y_1 = 2, y_7 = 3, y_{23} = 4, y_{38} = 3$ with objective value z=20 . In the first stage, it used the 1^{st} cutting pattern (4 times), the 5^{th} cutting pattern (12 times), and the 8^{th} cutting pattern (4 times). Otherwise, in the second stage, it used the 1^{st} cutting pattern (twice) in the first stripe, the 3^{rd} cutting pattern (3 times) in the second stripe, the 16^{th} cutting pattern (4 times) in the third stripe and the 15^{th} cutting pattern (3 times) in the fourth stripe.

For details, the summary of optimal solutions between the Gilmore and Gomory model and the set covering model can be seen in Table 6. From both of the models, the set covering model yields more patterns combination than the Gilmore and Gomory model. The more patterns combination created, the more stock used, and the trim loss will be bigger. But, the Gilmore and Gomory model uses less stock than the set covering model.

4. CONCLUSIONS

From the result and discussion, it can be concluded that the Gilmore and Gomory model and the set covering model can be implemented in the Cutting Stock Problem, especially in multiple stocks cases. Both of the models which were solved by LINGO 13.0 showed that some optimal cutting patterns used in the first stage can be reused in the second stage. There are more patterns combination generated from the set covering model rather than the Gilmore and Gomory model which means the Gilmore and Gomory model uses less stock and yields the minimum trim loss.

For further research, the more extensions of the dimensions in the Cutting Stock Problem are critically important to have models that are more realistic rather than previous models. Computational tests for further research are suggested.

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