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Research Paper

Chaos Theory of 0-1 Test and Logistic Map in New Confirmed COVID-19 Cases

Norliza Muhamad Yusof[1]*, Muhamad Luqman Sapini[1], Lidiya Irdeena Az'hari[2], Nor Akma Hanis Roslee[1], Siti Noor Afiqah Rahmat[1], Siti Hidayah Muhad Salleh[1]

[1]*Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Cawangan Negeri Sembilan, Seremban, 70300, Malaysia*
[2]*Jabatan Perangkaan Malaysia, Negeri Melaka Aras 7&8, Anyer Keroh, Melaka, 75450, Malaysia*
***Corresponding author**: norliza3111@uitm.edu.my

Abstract
This study aims to forecast the new number of positive cases of COVID-19 using the logistic map, where the presence of chaos is determined using the 0-1 Test. There are a small number of investigations on chaos utilizing the 0-1 Test and logistic map in the literature. This study provides a straightforward technique of forecasting and a current way of determining chaos. Information on new confirmed COVID-19 cases and population of ten countries involving China, Vietnam, Korea, Malaysia, Singapore, New Zealand, India, USA, Brazil, and Mexico for six months from January 20 to June 15, 2020, are selected as samples. The logistic map runs through three stages: data training, forecasting, and validation using the Mean Absolute Error method (MAE). The data training procedure is critical for determining the best growth rate r, for the logistic map. In chaotic investigations of the 0-1 Test, there appears to be an inverse expectation toward a logistic map. The 0-1 Test in the data of new confirmed COVID-19 cases in all the selected countries reveals the presence of non-chaotic. This contrasts with the existence of chaos in the logistic map forecasts for the USA, Brazil, and Mexico. Regardless, the logistic map was found to be capable of forecasting new COVID-19 positive cases with low error instances. Beginning in the middle of May 2020, new COVID-19 positive cases are forecasted to be on the rise in the USA, Brazil, and Mexico.

Keywords
Logistic Map, 0-1 Test, Chaos, Forecast, COVID-19

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1. INTRODUCTION

As time passes, scientists work backward using historical data and mathematical models to analyze and forecast how easily the virus spread. Several studies of mathematical modeling such as Susceptible-Infectious- Recovered (SIR), Susceptible-Exposed-Infectious-Recovered (SEIR), Susceptible-Exposed-Infectious-Hospitalized-Recovered (SEIHR) models, and other logistic models have been published to present and predict the dynamical behavior of COVID-19 (Sunhwa and Moran, 2020; Kamrujjaman et al., 2020; Postavaru et al., 2021; Shaikh et al., 2020; Sunthornwat and Areepong, 2021; Wang et al., 2020; Zeb et al., 2020). In this study, the spread of the COVID-19 is analyzed using chaos theory of 0-1 Test and logistic map.

According to Cartwright (1991), chaos theory is the study of nonlinear dynamical systems that are important in the unpredictability of activities with the emergence of different patterns. Chaos theory can be used in many fields of study. For instance, in forecasting weather (Slingo and Palmer, 2011), rainfall Sivakumar et al. (2001a), river system (Albostan et al., 2015; Ghorbani et al., 2010; Gao et al., 2019; Sivakumar et al., 2001b) , motorized traffic flow on urban road networks (Adewumi et al., 2016), and in COVID-19 outbreak (Jones and Strigul, 2021; Mangiarotti et al., 2020; Sapkota et al., 2021). According to Effah-Poku et al. (2018), the chaos theory must satisfy these three conditions; (1) the solution must be bounded, (2) the solution must be periodic, and (3) the solution must be exponentially sensitive to initial conditions. Lorenz experimented with weather forecasting 50 years ago, and he recognized that sensitivity to initial conditions is the reason for nonperiodic behaviour (Slingo and Palmer, 2011). It means that the more a method has the measurements to vary, the less expected it is to give a repeating sequence. Then, this sensitivity causes a very difficult forecasting process far in advance (Sapini et al., 2017). The presence of chaos can be detected by using the Correlation Dimension, 0-1 Test, Lyapunov Exponent, and Brock, Dechert, and Scheinkman Test (BDS).

The study of chaos can be described by the logistic map as chaos may exist in the growth rate parameter of the logistic map. A logistic map is defined as a chaotic map represents by a non-linear differential equation with a discrete timestep obtained by iteration map function (Ausloos, 2006). It is initially developed to predict population growth (Kendall and Fox, 1998; Lloyd, 1995) and has been used widely in the ecological model (Storch et al., 2017), cryptography (Moysis et al., 2020), data encryption (Huang and Ding, 2020; Moysis et al., 2020; Raghuvanshi et al., 2020). The application of a logistic map in the study of COVID-19 can be found in past studies (Mora et al., 2020; Koltsova et al., 2020; Pelinovsky et al., 2020). Simple mathematical structure and broad chaotic systems in Logistic map make the model widely used in many fields. We discovered that prior studies on analyzing the presence of chaos using the 0-1 Test and the usage of a logistic map in COVID-19 analysis have been limited.

Therefore, the logistic map is used in this work to forecast the new number of COVID-19 positive cases in selected countries after the presence of chaos is determined in the COVID-19 data using the 0-1 Test. The 0-1 Test is a novel approach in chaos theory that gives numerical values ranging from 0 to 1, making it easier to discern between chaotic and regular behavior. The Lyapunov Exponent is a classic method, while the Correlation Dimension and BDS methods are more suitable for financial data analysis.

2. EXPERIMENTAL SECTION

2.1 Materials

The data of new confirmed COVID-19 cases and the population was obtained from the website of Worldometers (2020). These daily statistics include countries affected by the pandemic as well as data that was accessible for 6 months at the time of the study. Therefore, data of 10 countries which are China, Korea, Malaysia, Singapore, Vietnam, New Zealand, Brazil, India, Mexico, and the United State of America (USA) is used from 20^{th} January 2020 until 15^{th} June 2020. Table 1 is the data descriptive of the data obtained.

2.2 Methods

2.2.1 The 0-1 Test

The 0-1 Test is a binary test used to study the dynamics of deterministic chaos of the time series data of a new number of positive cases of COVID-19. Maple Software was used to run the 0-1 Test to distinguish between normal dynamics and chaos exist in the data.

Suppose ϕ(j) is a time series data where j =1, ..., N is the number of data. Then, the following steps are performed (Gottwald and Laskar, 2016):

The first step is to calculate the following variables:

$$p_c(n) = \sum_{j=1}^{n} \varphi(j) cos(jc)$$

$$q_c(n) = \sum_{j=1}^{n} \varphi(j) sin(jc) \quad (1)$$

for cϵ (0, π) and n = 1, 2,..., N. The second step is to compute the mean square displacement, $M_c(n)$ as follows:

$$M_c(n) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} [p_c(j+n)-p_c(j)]^2+[q_c(j+n)-q_c(j)]^2 \quad (2)$$

The limit will be guaranteed by calculating $M_c(n)$ for $n \leq n_{cut}$ where $ncut \ll N$, and n_{cut} = N/10 to have better values of $M_c(n)$. The third step is to estimate the asymptotic growth rate, K_c using the following equation:

$$K_c = \lim_{N\to\infty} \frac{log M_c(n)}{log(n)} \quad (3)$$

The K_c is determined numerically by linear regression of log $M(n)$ versus log n. These three steps were repeated for other values of cϵ (0, π) selected randomly for 5 times by the system in the Maple Software. Lastly, the value of K is calculated by finding the median of K_c in Equation 3 as expressed below:

$$K = median(K_c) \quad (4)$$

If $K \approx 0$, then normal/regular dynamics are displayed, but if $K \approx 1$, chaotic dynamics are displayed. The 0-1 Test is utilized to determine the presence of chaos in the new confirmed COVID-19 cases as indicated by the value of K. This procedure is necessary to understand how the growth rate behaves when the logistic map is used. If chaotic was discovered, an unstable growth rate was obtained, causing the forecasting to become chaotic. However, if a regular pattern is discovered, a stable growth rate is obtained, causing the forecasts to converge to a specific number.

2.2.2 Logistic Map Model

The logistic map model is a non-linear function of degree two, frequently referring to a standard example of how complex, chaotic behaviour can take place from primary nonlinear dynamic equations (Tiwari and Hamsapriye, 2018). The logistic map is used to forecast the new number of COVID-19 positive cases for selected countries. This is expressed mathematically as (Akter and Chowdhury, 2018):

$$X_{n+1} = rX_n(1 - X_n), n = 0, 1, 2, ...N \quad (5)$$

In this study, the parameter $X_n \in [0,1]$ is a normalized ratio of the new confirmed cases to the maximum possible population at day-n. Meanwhile $r \in [0,4]$ is the growth rate of new COVID-19 positive cases calculated based on iteration approach.

Before the forecasting procedure takes place, data training was carried out using 70% of the data acquired for X_n, that is data from 20^{th} January 2020 to 2^{nd} May 2020. The process

Table 1. The Data Descriptive

Descriptive	N	Minimum	Maximum	Mean	Std. Deviation
New confirmed COVID-19 cases	1480	0	39072	2480	6792.465
Population	1480	4822233	1439323776	368346932	529692596.296
Valid N (listwise)	1480				

of selecting the lowest error of r is known as data training. Iteration approach is used to select the lowest error of r, where the r values were tested repeatedly from 0.1 to 4.0 in the direction of Equation 5. In the meantime, the Mean Absolute Error (MAE) of the anticipated and actual values of $X_n + 1$ is calculated to find the r-value with the least amount of error, up to 10-3. Then, the forecasting procedure follows. This time, the remaining 30% of data, that is, data from 3^{rd} May to 15^{th} June 2020, is utilized to forecast the values of $X_n + 1$ by substituting the selected r into Equation 5. The forecasted new number of positive COVID-19 cases is then calculated by multiplying the value of $X_n + 1$ obtained by the number of populations in the previous day. To determine the correctness of the forecasted values, the MAE of the forecasted and actual of $X_n + 1$ is used once more. This formula from Ahmar (2020) is used to determine the MAE:

$$MAE = \frac{\sum_n^N |Y_n - Z_n|}{N}, n = 0, 1, 2, ...N \quad (6)$$

where Y_n is the forecasted and Z_n is the actual data for day-n. Figure 1 shows the steps taken to implement the logistic map.

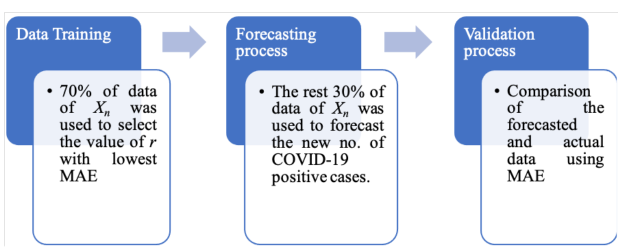

Figure 1. Data implementation Process in Logistic Map

3. RESULTS AND DISCUSSION

The 0-1 test is used to determine the presence of chaos in the data of new confirmed COVID-19 cases from January 20 to May 2, 2020. Chaos is determined by the K-values calculated from Equation 4 following to the processes outlined in section 3.2.1 using Maple software. Table 2 displays the results.

Table 2 depicts the dynamical behaviour of new COVID-19 positive cases in ten countries. If K is larger than 0.9, the presence of chaos in the data is determined (Sapkota et al., 2021). It appears that all countries have K-values smaller than 0.9, and thus, regular dynamics are depicted. These findings

Table 2. The Dynamical Behavior of a New Number Of COVID-19 Positive Cases for 10 Countries

Country	K	Dynamical Behaviour
USA	0.17	Regular
Korea	0.27	Regular
Malaysia	0.37	Regular
Singapore	0.46	Regular
New Zealand	0.59	Regular
Mexico	0.66	Regular
China	0.71	Regular
India	0.75	Regular
Vietnam	0.84	Regular
Brazil	0.89	Regular

are backed up by Sapkota et al. (2021), which found the occurrence of non-chaotic in all countries except Mexico, China, Vietnam, and Brazil. Hence, the data of COVID-19 is said to be non-chaotic, stable, and unaffected by initial conditions. Based on this conclusion, the forecasting trajectories of new COVID-19 positive cases for the ten countries are predicted to converge in the future.

In the meantime, the study additionally used a logistic map to forecast the new number of COVID-19 positive cases in the selected 10 countries from May 3 to June 15, 2020. As explained in section 3.2.2, the growth rates of the new COVID-19 positive cases, r with the lowest MAE of each country are obtained initially using the iteration approach under the data training process. Then, the selected r is used to forecast the new number of COVID-19 positive cases using Equation 5. Lastly, the accuracy of the forecasts is determined using the MAE shown in Equation 6. The findings on the growth rates, r of the ten countries are reported in Table 3.

Table 3. The Lowest MAE of r for 10 Countries

Country	Growth rate, r	Mean Absolute Error (MAE)
Vietnam	0.560	1.99E-08
New Zealand	0.773	1.8322E-06
China	0.851	0.00000024
Korea	0.860	5.671E-07
Singapore	0.865	8.8815E-06
Malaysia	0.940	0.00000059
India	0.986	6.58E-08
USA	1.048	3.6104E-06
Mexico	1.063	2.946E-07
Brazil	1.066	0.000000256

Meanwhile, Figure 2 through 11 show the forecasted and actual new numbers of positive COVID-19 cases combined into one graph. The errors between the forecasted and actual new numbers of positive COVID-19 cases are shown in Table

4.

Table 4. The MAE of The Forecasted and The Actual X_{n+1} for 10 Countries

Country	Mean Absolute Error (MAE)
Vietnam	1.49E-08
New Zealand	9.18E-08
China	5.36E-09
Korea	5.80E-07
Singapore	7.23E-05
Malaysia	1.04E-06
India	3.69E-06
USA	2.43E-04
Mexico	3.96E-05
Brazil	1.14E-04

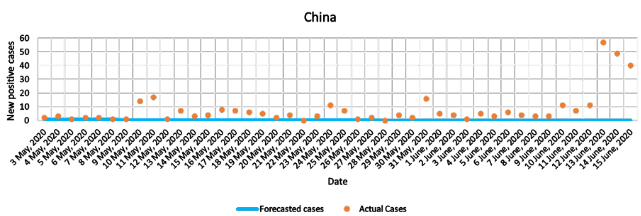

Figure 2. The Forecasted and Actual New Number of Positive Cases of COVID-19 in China

Table 3 presents the growth rate, r with the lowest MAE for 10 countries. The growth rate represents the expected rate of infection with COVID-19. Vietnam, for example, has the lowest r, where the expected rate of infection with COVID-19 is 0.56 per unit of population. Meanwhile, Brazil has the highest r with the expected rate of infection with COVID-19 is 1.0663 per unit of population. The USA, Mexico, and Brazil are three countries that are projected to have a higher chance of having their citizens become infected with COVID-19. The r-values of these countries are all greater than one. This also suggests that the forecasted new number of COVID-19 positive cases will be higher. In the remaining countries with r-values of less than one, the number of positive COVID-19 cases is expected to drop, with zero cases possible.

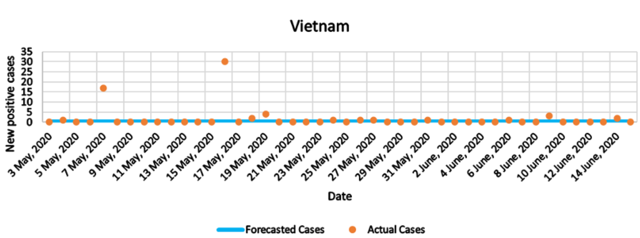

Figure 3. The Forecasted and Actual New Number of Positive Cases of COVID-19 in Vietnam

According to Figures 2 and 3, the new number of COVID-19 positive cases in China and Vietnam is forecasted to be zero. These forecasted values appear to be in line with the actual data from Vietnam, rather than China. In China, the number of new COVID-19 positive cases is increasing and declining in a cyclical pattern.

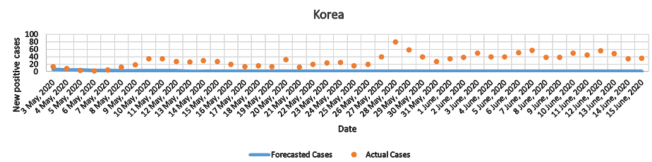

Figure 4. The Forecasted and Actual Number of Positive Cases of COVID-19 in Korea

Meanwhile, the new numbers of COVID-19 positive cases are forecasted to decline in the countries of Korea, Malaysia, Singapore, India, and New Zealand. We found that the graph curves between the forecast and the actual data are not parallel to each other. Yet, the actual numbers of positive COVID-19 cases are still reported to be decreasing in those countries, except for Korea and India. This can be seen from Figures 4, 5, 6, 7 to 8, where the actual new COVID-19 positive cases in Korea and India are increasing over the periods.

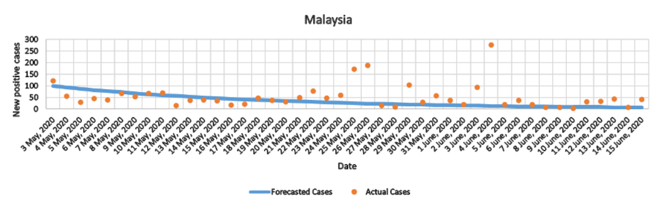

Figure 5. The Forecasted and Actual New Number of Positive Cases of COVID-19 in Malaysia

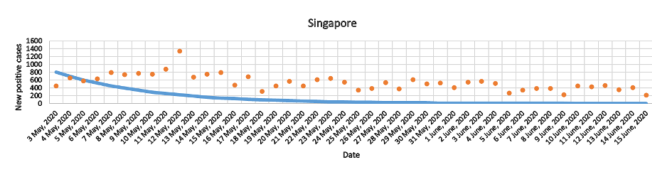

Figure 6. The Forecasted and Actual New Number of Positive Cases of COVID-19 in Singapore

In countries such as the USA, Brazil, and Mexico, the new numbers of COVID-19 positive cases are forecasted to rise. The increases are correlated with the actual new number of positive COVID-19 cases reported in Brazil, but not in the USA or Mexico, as shown in Figures 9 through 11. The graph of the actual data for the USA exhibits a cyclical pattern, while for Mexico, a horizontal movement is shown.

The new numbers of COVID-19 positive cases forecasted from Figures 2 until 11 overall follow the growth rates, r measured in Table 3. Those countries that have r-values less than

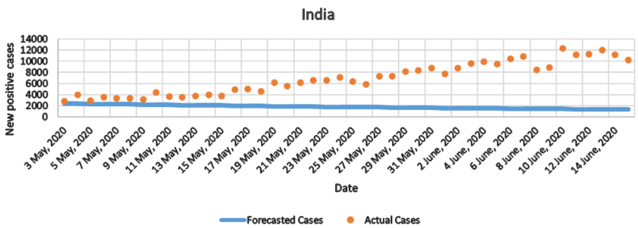

Figure 7. The Forecasted and Actual New Number of Positive Cases of COVID-19 in India

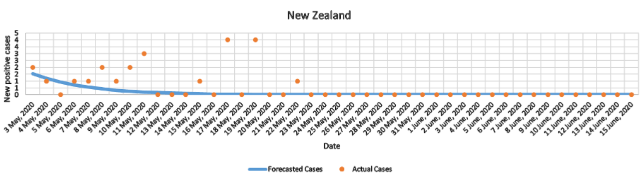

Figure 8. The Forecasted and Actual New Number of Positive Cases of COVID-19 in New Zealand

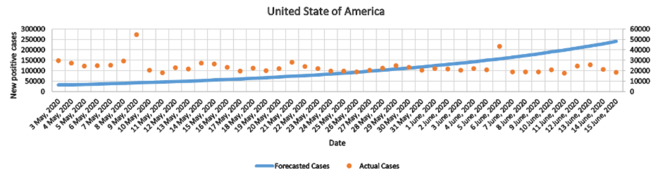

Figure 9. The Forecasted and Actual New Number of Positive Cases of COVID-19 in the USA

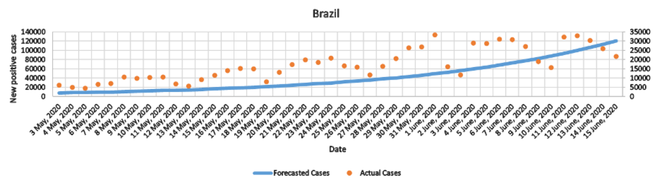

Figure 10. The Forecasted and Actual New Number of Positive Cases of COVID-19 in Brazil

one have a higher potential to reduce their new number of COVID-19 positive cases compared to countries that have r-values greater than one, such as the USA, Brazil, and Mexico. The new numbers of COVID-19 positive cases are forecasted to grow in these three countries.

Based on the graphs shown in Figures 2 through 11, it is difficult to measure the accuracy of the forecasting, and thus, the validation process yielded the last result, which is shown in Table 4. From May 3 through June 15, 2020, the forecasted new number of positive COVID-19 cases was verified by calculating the MAE between the forecasted and actual $X_n + 1$. All the predicted MAEs are tiny, with errors ranging from 10-4 to 10-9, indicating that this is a reliable forecast.

We also discovered consistency and inconsistency following the results obtained from the 0-1 Test and logistic map model. The forecasted trajectories of the logistic map presented in Figures 2 through 8 are found to be well-matched to the dynamical behaviour described by the 0-1 Test in Table 2. In particular, the forecasted trajectories are found to have converged to a number as predicted in the data's regular dynamic. Nevertheless, the forecasted trajectories in Figures 9 through 11 are found chaotic, where there are increments shown in the graphs, indicating an inconsistency with the data's regular dynamic behavior. As further data was considered, we discovered that new cases in the USA, Brazil, and Mexico were indeed growing until early 2021, as reported by the World Health Organization.

4. CONCLUSIONS

This study illustrates the methodology and outcomes of using the logistic map model and the 0-1 Test. The first type of data used is the new number of COVID-19 positive cases, and the second is the total population of the selected countries. The data was employed in the 0-1 Test to determine the data's dynamical behavior, and a logistic map was used to forecast the new number of COVID-19 positive cases. The growth rate and the normalized ratio of new positive COVID-19 cases to the overall population are two parameters evaluated in the logistic map. The following are the findings of this research.

The 0-1 Test shows that the data of the new number of COVID-19 positive cases for all selected countries contains non-chaotic cases. However, the forecasted trajectories of the logistic map for the USA, Brazil, and Mexico are found increasing as reported by the World Health Organization (WHO). The occurrence of chaos is detected in the forecasts of the logistic map but not in the 0-1 Test. According to Sapkota et al. (2021), this may be due to the geographic size and massive population of those countries that makes the 0-1 Test failed to determine the dynamical behaviour in the data of COVID-19 positive cases. Therefore, the logistic map is found to be more reasonable than the 0-1 Test in evaluating the presence of chaos in COVID-19 data for certain large countries such as the USA, Brazil, and Mexico.

In addition, COVID-19 positive cases are found rare in most countries with modest growth rates computed from a logistic map, but they are prevalent in countries with growth rates of more than one. For China, Vietnam, Malaysia, Singapore, New Zealand, and Brazil, the dynamic of new COVID-19 positive cases is forecasted to roughly match the actual increment and decrement. Accordingly, the logistic map is said to be a reliable model with low MAE.

In conclusion, the logistic map model used provides useful information for controlling the spread of the COVID-19 pandemic. More research into the dynamical behaviour of COVID-19 data is required to determine the presence of chaos. Future research could investigate the differences between some of the conclusions gained from the logistic map and the chaos theory implied in the 0-1 Test. It should also be highlighted those numerous other factors affecting the propagation of the COVID-19 must be considered, including the economy, social

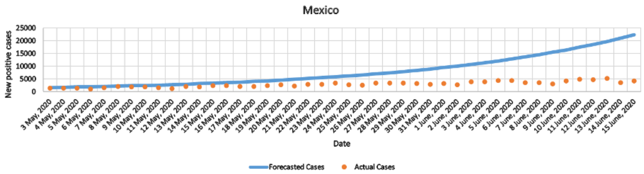

Figure 11. The Forecasted and Actual New Number of Positive Cases of COVID-19 in Mexico

considerations, and political concerns.

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