

Improve Fuzzy Inventory Model of Fractal Interpolation with Vertical Scaling Factor

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Abstract

The inventory model is used to determine the optimal inventory of a product. In certain cases, parameters in the inventory model are uncertain. Fractal interpolation techniques can be used to overcome parameter with uncertainty. Fractal interpolation results are affected by the fractal interpolation function and the vertical scaling factor. The vertical scaling factor is positive and less than 1. In this study, fractal interpolation techniques are introduced with variations in vertical scaling factor to overcome the uncertainty of demand data in inventory models. Furthermore, the interpolation results are used in fuzzy inventory models and expressed by Trapezoidal Fuzzy Number. This paper considers an inventory model with varying demand to optimize rice inventory. Based on the data obtained, the accuracy level will increase for the vertical scaling factor values close to 1. Optimal rice inventory of each successive fuzzy parameter is 1447963, 1013914, 504950, 215312. If the cost parameter is increased, then the amount of inventory is decreases.

Keywords

Fractal Interpolation, Vertical Scaling Factor, Fuzzy, Inventory Model

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1. INTRODUCTION

Interpolation is a technique for determining a value that is between two known values. There are two interpolation techniques, namely numerical interpolation and fractal interpolation. Several numerical interpolation techniques are Spline interpolation (Geng and Wu, 2021), Cubic-B-Spline interpolation (Geng and Wu, 2021), Multi-level Quadratic Spline interpolation technique (Geng and Wu, 2021), Newton Polynomial interpolation (Geng and Wu, 2021), Multi-variable Newton Polynomial interpolation (Geng and Wu, 2021), and interpolation Lagrange (Huang et al., 2019). In contrast to numerical interpolation, in fractal interpolation the interpolation process is carried out by constructing a fractal interpolation function (FIF). FIF can be constructed from affine functions Navascues et al. (2020) and non-affine functions Băicoianu et al. (2021), (Ri, 2017). Research related to fractal interpolation was developed on the determination of FIF and vertical scale factor (Ri, 2018; Ri et al., 2021; Balasubramani, 2017; Ri, 2020; Tyada et al., 2021). In this study, fractal interpolation was carried out with several variations in the values of the vertical scaling factor. The vertical scaling factor given is in the interval (0,1). The accuracy of the interpolation results is measured using the Mean Absolute Percentage Error (MAPE). It will be seen whether there is an effect of changing the vertical scaling

factor on the MAPE value obtained. Python programming was developed for interpolation and graph visualization.

The application of interpolation has been widely carried out in various fields. Fractal interpolation in the analysis of Covid 19 cases (Păcurar and Necula, 2020), seismic trace analysis (Ochoa et al., 2020), and navigation systems (Han et al., 2021). In this study, fractal interpolation will be applied to inventory optimization problems. In the inventory problem there are several influencing factors, including demand, inventory costs and lead time. In certain types of products, the level of demand depends on the price. Therefore we need an approach technique to determine the value of the demand parameter. Generally, forecasting can be done if the data to be predicted is time series data (Russel et al., 2023). Forecasting techniques can also be used to define demand variables in inventory models with uncertainty (Prak and Teunter, 2019). Research Tiacci and Saetta (2009) provides an analysis of the impact of applying forecasting techniques to inventory problems. A statistical approach can also be applied if the data follows a certain probability distribution (Adeyemi et al., 2021). The forecasting technique carried out aims to determine the parameter value approach based on time analysis. However, if we want to analyze the change in demand based on price, then the interpolation technique is used. In this study, fractal interpolation techniques are

introduced to determine the value of the price-based demand data approach. Furthermore, the interpolated demand data is a parameter in the inventory model with uncertainty. The Economic Order Quantity (EOQ) model can be used for inventory optimization problems (Liao and Li, 2021; Shaikh et al., 2019; Çalıřkan, 2021; Thinakaran et al., 2019). Interpolated data is an approximate value so that the fuzzy approach that can be used. An inventory model with fuzzy parameters is called a fuzzy inventory models (Tahami and Fakhravar, 2020; Lisan, 2018; Rani et al., 2019; Shee and Chakrabarti, 2021; Laura, 2020). Fuzzy parameters can be expressed by Trapezoidal Kalaiarasi et al. (2020a), Pentagonal (Onyenike and Ojarikre, 2022), and Hexagonal (Chakraborty et al., 2021), Triangular fuzzy number (Susanti et al., 2019). The EOQ model with fuzzy parameters is called the fuzzy EOQ models (Hemalatha and Annadurai, 2023). In this study, trapezoidal fuzzy numbers were used as fuzzy parameters for the EOQ fuzzy model. In this paper, interpolation techniques and fuzzy EOQ models are applied to rice supply planning with the aim of minimizing inventory costs.

Table 1. MAPE Value Criteria

MAPE (%)	Criteria
<10%	Very Good
0% ≤ MAPE < 20%	Good
20% ≤ MAPE ≤ 50%	Sufficient
>50%	Poor

Table 2. MAPE Value

Vertical Scaling Factor	MAPE (%)
0.1	32.85
0.2	29.84
0.3	26.82
0.4	23.80
0.5	20.79
0.6	17.77
0.7	14.75
0.8	11.73
0.9	8.72

2. EXPERIMENTAL SECTION

2.1 Research Method

In this research, interpolation techniques are applied to inventory problems with parameter uncertainty. The interpolation technique used is fractal interpolation. Fractal interpolation technique is used to determine the parameter values of fuzzy demand in inventory model. The research stage is to interpolate fractals with vertical scaling factor variations. From this stage, the approximate values for the fuzzy demand parameters

are obtained. The next step is to determine the optimal rice supply and total cost using the fuzzy EOQ model. There are two concepts used in this study, the first is fractal interpolation and the second is inventory. The stages of interpolation and solving inventory problems are given as follows.

2.2 Fractal Interpolation

One of the stages of fractal interpolation is building FIF. In this research, FIF is built from affine functions. We will see the effect of the interpolation results on changes in the value of the vertical scaling factor. In the following, the steps for carrying out fractal interpolation are given (Raubitzek and Neubauer, 2021). Given data $(x_i, y_i) \in \mathbb{R}^2; i=0, 1, \dots, N, x_0 < x_1 < \dots < x_N$. Based on the data in Step 1, it is determined that FIF is an affine transformation

$$\mathbb{R}^2; W_n, n=1, 2, \dots, N$$

$$W_n \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_n & 0 \\ c_n & S_n \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_n \\ e_n \end{bmatrix} \tag{1}$$

which is constrained to satisfy

$$W_n \begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} x_{n-1} \\ y_{n-1} \end{bmatrix} \text{ and } W_n \begin{bmatrix} x_N \\ y_N \end{bmatrix} = \begin{bmatrix} x_N \\ y_N \end{bmatrix} \tag{2}$$

For every $n=1, 2, \dots, N$

$$a_n = \frac{x_n - x_{n-1}}{x_N - x_0} \tag{3}$$

$$d_n = \frac{x_N x_{n-1} - x_0 x_n}{x_N - x_0} \tag{4}$$

$$c_n = \frac{y_n - y_{n-1}}{x_N - x_0} - S_n \frac{y_N - y_0}{x_N - x_0} \tag{5}$$

$$e_n = \frac{x_N - y_{n-1} - x_0 y_n}{x_N - x_0} - S_n \frac{x_N y_0 - x_0 y_N}{x_N - x_0} \tag{6}$$

Here, the real numbers a_n, d_n, c_n, e_n are determined by the interpolation points and S_n is an independent parameter referred to as the vertical scaling factor. In this paper, the stages of interpolation and graph visualization use Python programming. The level of accuracy of the interpolated data results can be determined using the MAPE. The MAPE formulation is given as follows.

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{y_t - \hat{y}_t}{y_t} \right| 100\% \tag{7}$$

Where

- n : amount of data
- y_t : actual data in period t
- \hat{y}_t : interpolation data result in period t

Table 3. Demand Data Interpolation Result

	Demand Interpolation with Vertical Scaling Factor 0.1 (Kilogram)	Demand Interpolation with Vertical Scaling Factor 0.9 (Kilogram)
	2591763	2138331
	2849792	1715265
	1177733	1177733
	866987	1201686
	2419069	2419069
	362400	362400
	2811013	2150633
	2588790	2111570
	2633104	1726022
	596369	822785.9
	2742190	2315604
	3138678.5	3138678.5

Table 4. Fuzzy Parameter Inventory Model

Parameter	Fuzzy Parameter
k	(250; 450; 650; 1050)
a	(362400; 1034702; 2511928; 3138679)
b	(11.287; 12.108; 26.604; 50)
P	(1.181; 1.148; 1.245; 1.249)
c	(9200; 9400; 9600; 9800)
g	(28.92; 29.86; 35.06; 38.30)

2.3 Inventory Model

Inventory model is used to determine the optimal amount of inventory so that total costs are minimized. One of the inventory models that can be used is the economics order quantity model (EOQ). Optimal inventory is influenced by several factors including demand, costs, lead time, and others. In certain cases, the inventory model parameters have fluctuating values and cannot be determined, so a fuzzy approach can be used to overcome parameter uncertainty. In this article, an interpolation technique is introduced to overcome the uncertainty of inventory parameters. The inventory model used is the EOQ fuzzy model [Kalaiarasi et al. \(2020a\)](#) where the demand data parameters are defined based on the interpolated. Given Trapezoidal fuzzy number $\tilde{B} (n_1, n_2, n_3, n_4)$. The graded mean integration method is used in the defuzzification stage so that it is obtained ([Kalaiarasi et al., 2020b](#)).

$$P(\tilde{B}) = \frac{n_1 + 2n_2 + 2n_3 + n_4}{6} \tag{8}$$

Based Equation (8), the following is given a fuzzy EOQ model introduced by obtained ([Kalaiarasi et al., 2020b](#)).

$$JTC(Q) = 1/6 \left[\left(\frac{k_1(a_1 - b_1P_1)}{Q_4} c_1(a_1 - b_1P_1) + \frac{g_1c_1Q_1}{2} \right) + 2 \left(\frac{k_2(a_2 - b_2P_2)}{Q_3} c_2(a_2 - b_2P_2) + \frac{g_2c_2Q_2}{2} \right) + 2 \left(\frac{k_3(a_3 - b_3P_3)}{Q_2} c_3(a_3 - b_3P_3) + \frac{g_3c_3Q_3}{2} \right) + \left(\frac{k_4(a_4 - b_4P_4)}{Q_1} c_4(a_4 - b_4P_4) + \frac{g_4c_4Q_4}{2} \right) \right] \tag{9}$$

$$Q_1 = \sqrt{\frac{2k_4(a_4 - b_4P_4)}{g_1c_1}} \tag{10}$$

$$Q_2 = \sqrt{\frac{2k_3(a_3 - b_3P_3)}{g_2c_2}} \tag{11}$$

$$Q_3 = \sqrt{\frac{2k_2(a_2 - b_2P_2)}{g_3c_3}} \tag{12}$$

$$Q_4 = \sqrt{\frac{2k_1(a_1 - b_1P_1)}{g_4c_4}} \tag{13}$$

Where

- k : ordering cost (rupiah)
- a : constant demand rate coefficient (kilogram)
- b : price dependent demand rate coefficient (rupiah)
- P : selling price (rupiah)
- Q_i : Ordering size , $i=1,2,3,4$ (kilogram)
- c : unit purchasing cost (rupiah)
- g : constant holding cost coefficient (rupiah)

Table 5. Inventory Optimal Solution

Q ₁	Q ₂	Q ₃	Q ₄	JTC
1447963	1013914	504950	215312	138295057481

3. RESULTS AND DISCUSSION

In this section, fractal interpolation is applied to rice demand data at the Regional Division of BULOG, South Sumatra and Bangka Belitung Regional Division. The type of rice used is a commercial type. Rice demand data is interpolated based on rice price estimates. Furthermore, the interpolated demand data is used as a fuzzy parameter in the fuzzy EOQ model. The fuzzy EOQ model is used to determine the optimal rice inventory so that the total cost of inventory can be minimized.

3.1 Fractal Interpolation with Vertical Scaling Factor

Given the initial condition of the fractal interpolation. (10000,3 138679.5), (10800,1177733), (11400,2419069), (12000,3624 00). The Fractal interpolation was performed for successive vertical scaling factors 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9 and the process uses (1) to (6). Visualization of the interpolation graph for each scaling factor is given in the following graph.

Figure 1 illustrates the interpolated demand for commercial rice and the actual data based on price. The blue line is for actual data demand and the yellow line is for interpolation results. The lowest price is Rp.10000 and the highest price is Rp.12000. In Figure 1 (i), a MAPE value of 8.72 is obtained and based on Table 1 it is included in the very good category. This means that based on the data, the highest level of accuracy is obtained when the vertical scaling factor is 0.9. The level of accuracy using MAPE for each scaling factor is given in Table 2.

Based on Table 2, the highest level of accuracy is obtained when the scaling factor is equal to 0.9. Accuracy values were obtained for each different vertical scaling factor. Based on rice demand data at the Regional Division of BULOG South Sumatra and Bangka Belitung Regional Division, if the value of the vertical scaling factor is close to one, the accuracy level will be higher. This can be seen from the smaller MAPE value. Furthermore, the demand fuzzy parameters in the inventory models are taken from interpolation with a vertical scaling factor equal to 0.9. As a comparison of the interpolation results, the interpolation of demand data with the lowest and highest levels of accuracy is given in Table 3.

3.2 Fuzzy EOQ Model with Trapezoidal Fuzzy Numbers

Fuzzy parameters are expressed by Trapezoidal fuzzy numbers. Fuzzy parameters for demand data are determined based on interpolation results while other parameters are determined based on actual data. Fuzzy parameter values for demand data are determined based on the results of fractal interpolation with a vertical scaling factor of 0.9 where that value is determined by considering the highest value, mean, standard deviation

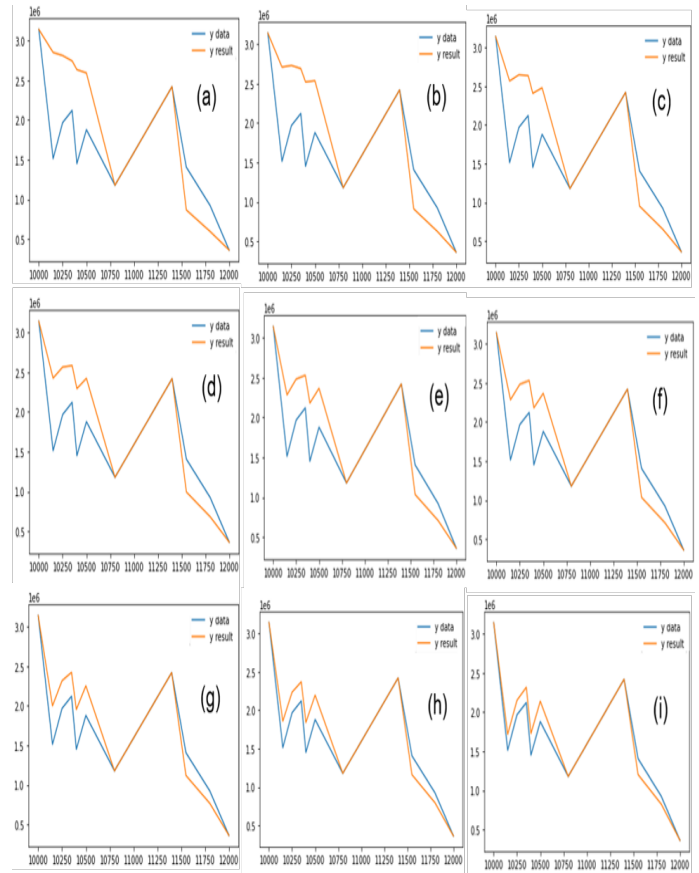


Figure 1. Fractal Interpolation with Vertical Scaling Factor $S_n=0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9$ in (a), (b), (c), (d), (e), (f), (g), (h), (i), Respectively

and lowest value. Determining the value of fuzzy parameters depends on the decision-maker. The application of the concept of descriptive statistics can be a consideration for decision-makers in determining the value of fuzzy parameters.

The fuzzy parameters in Table 4 are transformed into crips using (8) The total cost is determined using Equation (9) The optimal inventory is determined based on the Equation (10) to (13) and the optimal solution is given in Table 5.

There are four values in the trapezoidal fuzzy number. The optimal rice demand for each fuzzy parameter is obtained. Based on the data, the result is that the greater the value of the fuzzy parameter, the smaller the optimal inventory amount.

4. CONCLUSION

Based on the results of fractal interpolation and the fuzzy EOQ model, it can be concluded that for positive vertical scaling factors and less than one, the accuracy level will be increase for the scaling factor close to one. The optimal rice of inventory will be decreased for fuzzy parameter values that are increasing. This relates to the concept that inventory is related to costs. In this study, the focus is on discussing changes in the value of the scaling factor on the accuracy of the fractal interpolation technique. The value of the fractal interpolation results is influenced by the scaling factor and FIF. Further, research can be focused on changes in the form of FIF.

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