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Research Paper

Optimizing Generalized Linear Models for Real Estate Investment Risk: A Hybrid Genetic Algorithm and IRLS Approach

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Abstract
Generalized Linear Models (GLMs) serve as a powerful extension of traditional linear regression, enhancing statistical analysis capabilities. This study employs a GLM to forecast the Return on Investment (ROI) within Nigeria's real estate and construction sectors, utilizing a comprehensive dataset that includes data from the Nigerian Stock Exchange (NSE) and macroeconomic indicators from the National Bureau of Statistics (NBS) and the Central Bank of Nigeria (CBN), spanning 2017 to 2023. The model integrates critical macroeconomic indicators, including Gross Domestic Product (GDP) growth, the Consumer Price Index (CPI), Interest Rates (IR), and the Unemployment Rate (UR), while also considering interaction effects and non-linear terms to enhance predictive accuracy. Two methods were used to assess the GLM: Genetic Algorithms (GLM-GA) and Iteratively Reweighted Least Squares (GLM-IRLS), both of which revealed significant insights. Adjusted R^2 values ranged from 0.68 to 0.73, with the highest in 2019 (0.73) and the lowest in 2021 (0.68). The Bayesian Information Criterion (BIC) exhibited variation, with values between 365.50 in 2019 and 386.30 in 2021, indicating differing model efficiency across years. Investment risk metrics, such as Value at Risk (VaR) and Conditional Value at Risk (CVaR), showed upward trends, with VaR increasing from 7.50 in 2017 to 9.20 in 2022, and CVaR rising from 7.00 to 8.70 in the same period, reflecting heightened risk exposure. The findings underscore the sensitivity of ROI predictions to macroeconomic conditions and highlight the challenges of forecasting amid economic volatility. This study emphasizes the substantial impact of macroeconomic factors on ROI and the necessity of considering these variables in investment risk assessments.

Keywords
Generalized Linear Model, Return on Investment, Macroeconomic Indicators, Nigerian Stock Exchange, Investment Risk

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1. INTRODUCTION

Investment in emerging markets, especially within the real estate and construction sectors, offers lucrative opportunities but is accompanied by substantial risks that require sophisticated analytical methods to assess accurately (Lynn, 2010; Ametefe et al., 2023). The Nigerian real estate and construction industries have seen remarkable growth over the past two decades, driven by factors such as population expansion, rapid urbanization, and government infrastructure initiatives. This growth has attracted significant domestic and international investment, positioning the sector as a cornerstone of Nigeria's economic development (Olanrewaju et al., 2020; Abubakar et al., 2024a; Abdullahi and Bala, 2018). However, the sector is also plagued by challenges such as economic volatility, regulatory uncertainty, and fluctuating macroeconomic conditions, all of which complicate the accurate assessment of investment risk and the prediction of return on investment (ROI) (Moore, 2019; Abubakar et al., 2024b).

Traditional risk assessment methods often rely on linear models that assume a straightforward, proportional relationship between input variables and outcomes (Tziralis et al., 2009; Junkes et al., 2015). While these models can be useful in certain contexts, they are often inadequate for capturing the intricate, nonlinear interactions that characterize the economic environment of emerging markets like Nigeria. In such markets, the dynamics are far more complex, with multiple variables interacting in ways that are neither simple nor linear.

Key macroeconomic indicators such as Gross Domestic Product (GDP), interest rates (IR), unemployment rates (UR), and exchange rates (ER)-play crucial roles in shaping investment outcomes (Hasan et al., 2022). For example, fluctuations in GDP might have a varying impact on ROI depending on the current interest rate environment or changes in unemployment rates (Tudose and Avasilcai, 2021). Similarly, the effect of exchange rate volatility on investments could be amplified or mitigated by concurrent shifts in other economic factors, creating a web of interdependencies that are difficult to model accurately with traditional linear approaches (Chit et al., 2010; Mudiangombe and Mwamba, 2023; Naeem, 2024).

In the real estate and construction sectors, these nonlinear relationships are particularly pronounced (Bouchouicha and Ftiti, 2012). The market is highly sensitive to macroeconomic shifts, where small changes in one indicator can lead to disproportionate effects on investment returns (Kırdök, 2012). Conventional linear models, by their nature, fail to fully account for these complexities, often leading to oversimplified risk assessments that do not reflect the true volatility and uncertainty inherent in these markets (Alam and Ali, 2023). As a result, investors relying solely on traditional models may face significant challenges in accurately predicting ROI, potentially underestimating the risks involved or missing out on opportunities for optimization. The need for more sophisticated modeling approaches that can address these nonlinear interactions is evident, particularly in emerging markets where the economic landscape is constantly evolving and traditional models fall short of providing reliable forecasts.

To address the challenges in parameter estimation and model fitting, this research proposes an advanced methodology that integrates Generalized Linear Models (GLMs) with Genetic Algorithms (GAs) and Iterative Reweighted Least Squares (IRLS). By combining these approaches, the research aims to enhance the precision and robustness of parameter estimates. Specifically, GAs will be employed to optimize model parameters, navigating complex and multidimensional search spaces to improve predictive accuracy. Concurrently, IRLS will refine these estimates by iteratively adjusting weights to address outliers and heteroscedasticity, thereby ensuring more reliable and accurate model results.

The Generalized Linear Models (GLMs) are an extension of traditional linear regression models that offer greater flexibility in modeling a variety of data types and relationships. Their application spans numerous fields, demonstrating their versatility and importance:

In Public Health, GLM was employed to predict health outbreaks, aiming to reduce morbidity and mortality (Zareie et al., 2023). This model supports public health efforts by providing accurate forecasts that inform intervention strategies. In ecological studies, GLM was used to model species distribution based on environmental factors such as temperature and precipitation (Li et al., 2017). These models are crucial for understanding species-environment relationships and guiding conservation efforts. In finance, GLMs assess credit risk by evaluating the likelihood of loan defaults based on variables like borrower credit scores and income levels (Bunyaminu and Issah, 2012; İyit et al., 2021; Yılmaz, 2020). This helps financial institutions manage risk and make informed lending decisions. A study in political science used GLMs to study voting behavior by examining how demographic variables influence election outcomes (Traunmüller et al., 2015). GLMs predict consumer purchase decisions by modeling factors like price sensitivity and brand loyalty (Ivascu et al., 2022). This guides companies to optimize their marketing strategies and product offerings. Reliability Engineering, the GLMs model the time until system or component failure, incorporating factors such as stress and usage conditions, aiding in predicting maintenance needs and improving system design (Mahmood et al., 2022). These studies highlight the broad applicability of GLMs in providing data-driven information across various domains, demonstrating their critical role in accurate modeling and decision-making.

The Iterative Reweighted Least Squares (IRLS) is a robust technique used to enhance parameter estimation by addressing challenges such as outliers and heteroscedasticity. Its iterative approach to updating weights improves the accuracy and reliability of model estimates. Key studies illustrating the effectiveness of IRLS include: GLMs were used in logistic regression by adjusting weights to handle the variance in observations, improving the accuracy of maximum likelihood estimates (Hastie and Pregibon, 2017). IRLS is applied to reduce the influence of outliers on parameter estimates, enhancing model robustness. In quantile regression, IRLS was employed to iteratively adjust weights to focus on different quantiles of the response variable's distribution, providing detailed insights into variable relationships beyond mean regression models (Gad and Qura, 2016)). IRLS was extended to weighted least squares to address heteroscedasticity by reweighting observations based on their variance (Liu et al., 2014). The IRLS approach produced accurate parameter estimates in the presence of varying data dispersion (Marzjarani, 2019).

These studies reveal IRLS's capability to improve model robustness and accuracy by addressing common statistical issues, making it a valuable tool in both traditional and modern statistical modeling frameworks. Genetic Algorithms (GAs) are renowned for solving complex optimization problems, including parameter estimation and model fitting. Their ability to explore extensive search spaces and find near-optimal solutions makes them valuable in various applications. GAs have been applied induction motor parameter estimation using a genetic algorithm (Fortes et al., 2013). Estimating parameters in models where traditional methods may falter (Moiz et al., 2018; Idris and Muhammad, 2022). GAs have also been applied to parameter estimation in nonlinear dynamic systems characterized by chaotic behavior. Solving complex engineering design problems and efficiently handling multi-objective optimization (Bhoskar et al., 2015; Maghawry et al., 2021). GAs are applied to portfolio optimization and risk management, outperforming traditional methods in managing nonlinear and non-smooth objective functions common in financial models (Sang, 2021; Meier and Danzinger, 2022). These studies highlight the broad applicability and effectiveness of GAs across various fields, showcasing their role in enhancing parameter estimation, optimization, and decision-making processes.

This study aims to leverage GLM, GA, and IRLS to develop a robust model for assessing investment risk in Nigeria's real estate and construction sectors, with a focus on ROI as the primary dependent variable. By evaluating the effectiveness of this integrated approach compared to traditional linear models, the research will provide insights into how key macroeconomic indicators impact ROI and improve risk prediction accuracy.

The Nigerian real estate and construction sectors are crucial to the country's economic development but face significant investment risks due to economic volatility, regulatory changes, and varying macroeconomic conditions. Traditional linear models used for risk assessment often fail to capture the complex, nonlinear interactions between macroeconomic variables—such as GDP, IR, UR, and ER—and investment outcomes like ROI. This inadequacy can lead to inaccurate risk predictions and suboptimal investment decisions, highlighting the need for more sophisticated analytical techniques.

This research aims to develop a predictive model to enhance investment risk assessment in Nigeria's real estate and construction sectors by integrating Generalized Linear Models (GLM) with Genetic Algorithms (GA) and Iteratively Reweighted Least Squares (IRLS). The key contributions are:

1. Enhanced Predictive Model by combining GLM with GA and IRLS, the study offers a more accurate framework for modeling the nonlinear relationships between macroeconomic indicators (GDP, IR, UR, ER) and investment outcomes, improving risk prediction over traditional linear models.
2. Improved Parameter Estimation by utilizing Genetic Algorithms for optimization and IRLS for refining parameter estimates will enhance model robustness, addressing issues such as outliers and heteroscedasticity, and providing more reliable risk assessments.
3. The research will provide specific information into the market, identifying critical factors influencing investment risk and offering actionable guidance for both local and international investors.

This research addresses the critical need for advanced analytical tools in the assessment of real estate investment risks in emerging markets, offering a novel solution that improves the accuracy and reliability of risk assessments, ultimately supporting better investment decisions.

2. Materials and Methods

2.1 Generalized Linear Model (GLM)

In the GLM framework, the relationship between the explanatory variables and the response variable is characterized by a systematic component and a random component. The core formulation of a GLM is expressed in Equation (1) as follows:

$$g(\mathbb{E}[Y_i]) = \mathbf{X}_i \boldsymbol{\beta} \tag{1}$$

where $g(\cdot)$ represents the link function, transforming the expected value of the response variable $\mathbb{E}[Y_i]$ to the scale of the linear predictors. In this context, we use the *log link function*, which is particularly suitable for modelling ROI, ensuring non-negative predicted values presnted in Equation (2):

$$\log(\mathbb{E}[Y_i]) = \mathbf{X}_i \boldsymbol{\beta} \tag{2}$$

This implies that the expected ROI, $\mathbb{E}[Y_i]$, is modelled as in Equation (3):

$$\mathbb{E}[Y_i] = \exp(\mathbf{X}_i \boldsymbol{\beta}) \tag{3}$$

where Y_i denotes the ROI for the i^{th} observation, which serves as the dependent variable, $\mathbb{E}[Y_i]$ represents the expected ROI,$\mathbf{X}_i$ is a vector of explanatory variables for the i^{th} observation, encompassing macroeconomic indicators such as GDP Growth, Consumer Price Index (CPI), Interest Rate (IR), and Unemployment Rate (UR) and $\boldsymbol{\beta}$ is a vector of regression coefficients that quantify the impact of these explanatory variables on ROI.

The GLM for predicting ROI can be specified in Equation (4) as follows:

$$\begin{aligned}\log(\mathbb{E}[ROI_i]) =& \beta_0 + \beta_1 \text{GDP Growth}_i + \beta_2 \text{CPI}_i \\ &+ \beta_3 \text{IR}_i + \beta_4 \text{UR}_i\end{aligned} \tag{4}$$

This can be rewritten in terms of the expected ROI in Equation (5) as follows

$$\begin{aligned}\mathbb{E}[ROI_i] =& \exp(\beta_0 + \beta_1 \text{GDP Growth}_i + \beta_2 \text{CPI}_i \\ &+ \beta_3 \text{IR}_i + \beta_4 \text{UR}_i)\end{aligned} \tag{5}$$

where GDP Growth$_i$ represents the GDP growth rate for the i^{th} investment,CPI$_i$ indicates the Consumer Price Index for the i^{th} investment, IR$_i$ is the interest rate applicable to the i^{th} investment, UR$_i$ denotes the unemployment rate affecting the i^{th} investment, β_0 is the intercept term, representing the baseline ROI when all explanatory variables are zero and β_1, β_2, β_3, β_4 are the coefficients to be estimated, reflecting the sensitivity of ROI to changes in each explanatory variable.

To further improve the accuracy of parameter estimation in the GLM framework, two advanced techniques are employed: Iteratively Reweighted Least Squares (IRLS) refine parameter estimates by adjusting weights iteratively based on the residuals, allowing for better convergence and stability in models with non-constant variance. Meanwhile, Genetic Algorithms (GA) offer a robust optimization approach by mimicking the process of natural selection to explore a broader solution space and avoid local minima, leading to more accurate and reliable parameter estimates, especially in complex or non-linear models.

IRLS is applied to iteratively refine the estimation of regression coefficients. The updated parameter estimates $\boldsymbol{\beta}$ are obtained by solving the weighted least squares problem presnted in Equation (6) as follows:

$$\boldsymbol{\beta}^{(t+1)} = \arg\min_{\boldsymbol{\beta}} \left[\mathbf{W}^{(t)}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})\right]^T \mathbf{W}^{(t)}(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}) \quad (6)$$

where $\mathbf{W}^{(t)}$ is a diagonal weight matrix updated at each iteration t, reflecting the influence of residuals on the parameter estimation.

GA is utilized for global optimization of the regression coefficients. The objective function $f(\boldsymbol{\beta})$ is defined in Equation (7):

$$f(\boldsymbol{\beta}) = \sum_{i=1}^{n} [\exp(\mathbf{X}_i\boldsymbol{\beta}) - Y_i]^2 \quad (7)$$

GA evolves a population of candidate solutions through selection, crossover, and mutation to minimize this objective function. The detailed methodology for these parameter estimation techniques, including their implementation and comparative advantage has been presented in the next section.

2.2 Iteratively Reweighted Least Squares (IRLS) for GLM Parameter Refinement

The Iteratively Reweighted Least Squares (IRLS) method is used to refine parameter estimates obtained from preliminary fitting procedures. This technique iteratively updates parameter estimates in a Generalized Linear Model (GLM) framework to improve accuracy and handle data irregularities. The detailed IRLS procedure is as follows:

Step 1: Initialization

The IRLS process begins with initial parameter estimates $\boldsymbol{\beta}^{(0)}$. These estimates are typically derived from a preliminary model fitting, such as a basic GLM fit.

Step:2 Weight Calculation

In each iteration k, weights $W_i^{(k)}$ are computed based on the residuals from the GLM. The weight for the i^{th} observation is determined using the derivative of the link function g with respect to the expected value $\mathbb{E}[ROI_i]$. Specifically, the weight matrix $W^{(k)}$ for iteration k is defined in Equation (8) as follows:

$$W_i^{(k)} = \text{diag}\left(\left(\frac{\partial g(\mathbb{E}[ROI_i])}{\partial \mathbb{E}[ROI_i]}\right)^{-2}\right) \quad (8)$$

where diag$(\cdot)$ denotes a diagonal matrix with $W_i^{(k)}$ on the diagonal, $\frac{\partial g(\mathbb{E}[ROI_i])}{\partial \mathbb{E}[ROI_i]}$ represents the derivative of the link function g with respect to $\mathbb{E}[ROI_i]$.

For example, if the log link function is used, $g(\mathbb{E}[ROI_i]) = \log(\mathbb{E}[ROI_i])$ and $\frac{\partial g(\mathbb{E}[ROI_i])}{\partial \mathbb{E}[ROI_i]} = \frac{1}{\mathbb{E}[ROI_i]}$. Thus, the weight is given in Equation (9) as follow,

$$W_i^{(k)} = \mathbb{E}[ROI_i]^2 \quad (9)$$

$$\frac{\partial g(\mathbb{E}[ROI_i])}{\partial \mathbb{E}[ROI_i]} = \frac{1}{\mathbb{E}[ROI_i]} \quad (10)$$

Thus, the weight is given in Equation (11) as follows

$$W_i^{(k)} = \mathbb{E}[ROI_i]^2 \quad (11)$$

Step 3: Weighted GLM Estimation

With the updated weights, the GLM is fitted anew to the data. The weighted GLM estimation can be expressed in Equation (12):

$$g(\mathbb{E}[ROI_i]) = \mathbf{X}_i\boldsymbol{\beta}^{(k)} \quad (12)$$

where $\mathbf{X}_i$ is the vector of explanatory variables for the i^{th} observation and $\boldsymbol{\beta}^{(k)}$ denotes the parameter estimates at iteration k.

The weighted likelihood function for the GLM can be formulated in Equation (13):

$$\mathcal{L}_W(\boldsymbol{\beta}) = -\frac{1}{2}\sum_{i=1}^{n}\left[W_i^{(k)}\left(ROI_i - g^{-1}(\mathbf{X}_i\boldsymbol{\beta})\right)^2\right] \quad (13)$$

where $g^{-1}(\cdot)$ denotes the inverse link function.

The weighted GLM estimation involves solving the weighted least squares problem in Equation (14):

$$\hat{\boldsymbol{\beta}}^{(k)} = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} W_i^{(k)}(ROI_i - \mathbf{X}_i\boldsymbol{\beta})^2 \quad (14)$$

Step 4: Parameter Update

The updated parameter estimates $\boldsymbol{\beta}^{(k+1)}$ are computed by solving the weighted least squares problem. The update formula as in Equation (15):

$$\boldsymbol{\beta}^{(k+1)} = \left(\mathbf{X}^\top W^{(k)}\mathbf{X}\right)^{-1}\mathbf{X}^\top W^{(k)}\mathbf{Y} \quad (15)$$

where $\mathbf{X}$ is the matrix of explanatory variables,$W^{(k)}$ is the diagonal weight matrix for iteration k and $\mathbf{Y}$ is the vector of observed ROI values.

The weight matrix $W^{(k)}$ updates the residual variance, making the estimation robust to heteroscedasticity in Equation (16):

$$\hat{\sigma}_i^2 = \frac{1}{n}\sum_{i=1}^{n} W_i^{(k)}\left(ROI_i - \mathbf{X}_i\boldsymbol{\beta}^{(k)}\right)^2 \quad (16)$$

Step 5: Convergence Check

The iteration continues until convergence is achieved. Convergence is checked by monitoring changes in parameter estimates in Equation (17) as follows:

$$\|\boldsymbol{\beta}^{(k+1)} - \boldsymbol{\beta}^{(k)}\|_2 < \epsilon \tag{17}$$

where ϵ is a small positive threshold that specifies the desired accuracy. The convergence criteria ensure that parameter estimates stabilize and the solution is precise.

This approach, outlined in Algorithm 1, integrates optimization techniques, including the Iteratively Reweighted Least Squares (IRLS) algorithm for Generalized Linear Models (GLM), to refine parameter estimation. Algorithm 1 provides a robust framework for assessing investment risk in Nigeria's real estate and construction sectors, enhancing both predictive accuracy and decision-making.

Algorithm 1 IRLS for GLM for Parameter Refinement

Require: Initial parameter estimates $\boldsymbol{\beta}^{(0)}$, Data $\{(ROI_i, \mathbf{X}_i)\}_{i=1}^{n}$, Convergence threshold ϵ
Ensure: Refined parameter estimates $\boldsymbol{\beta}^*$
$k \leftarrow 0$
while not converged **do**
Weight Calculation:
for each observation i **do**
Calculate expected value: $\mathbb{E}[ROI_i] \leftarrow g^{-1}(\mathbf{X}_i \boldsymbol{\beta}^{(k)})$
Compute weight: $W_i^{(k)} \leftarrow \left(\frac{\partial g(\mathbb{E}[ROI_i])}{\partial \mathbb{E}[ROI_i]}\right)^{-2}$
end for
Form weight matrix: $W^{(k)} \leftarrow \text{diag}(W_1^{(k)}, W_2^{(k)}, \ldots, W_n^{(k)})$
Weighted GLM Estimation:
Compute new parameter estimates:

$$\boldsymbol{\beta}^{(k+1)} \leftarrow \left(\mathbf{X}^\top W^{(k)} \mathbf{X}\right)^{-1} \mathbf{X}^\top W^{(k)} \mathbf{Y}$$

Convergence Check:
if $\|\boldsymbol{\beta}^{(k+1)} - \boldsymbol{\beta}^{(k)}\|_2 < \epsilon$ **then**
Convergence achieved, exit loop
else
$k \leftarrow k + 1$
end if
end while
Output:
Return refined parameter estimates $\boldsymbol{\beta}^* \leftarrow \boldsymbol{\beta}^{(k+1)}$

2.3 GA for GLM Parameter Optimization

To refine the parameter estimates of the Generalized Linear Model (GLM), Genetic Algorithms (GA) are employed. GA is an evolutionary optimization technique inspired by natural selection principles. It is particularly useful for complex parameter estimation problems where traditional methods might fall short.

Step 1: Optimization Problem Formulation

The objective is to optimize the parameters $\boldsymbol{\beta}$ of the GLM to minimize the discrepancy between observed and predicted values of Return on Investment (ROI). The optimization problem is formulated in equation (18) as follows:

$$\min_{\boldsymbol{\beta}} \mathcal{L}(\boldsymbol{\beta}) \tag{18}$$

where $\mathcal{L}(\boldsymbol{\beta})$ is the objective function defined in Equation (19) as follow:

$$\mathcal{L}(\boldsymbol{\beta}) = \sum_{i=1}^{n} (ROI_i - \mathbb{E}[ROI_i])^2 \tag{19}$$

In this study $\mathbb{E}[ROI_i]$ is the expected ROI predicted by the GLM and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)^T$ represents the vector of regression coefficients to be estimated.

The objective function $\mathcal{L}(\boldsymbol{\beta})$ measures the sum of squared errors between observed and predicted ROIs, which the GA aims to minimize.

Step 2: Genetic Algorithm Procedure

The GA follows a structured evolutionary process to optimize $\boldsymbol{\beta}$. The steps are detailed as follows:

Step 3: Initialization: Create an initial population of parameter vectors $\boldsymbol{\beta}^{(0)} = \{\boldsymbol{\beta}_1^{(0)}, \boldsymbol{\beta}_2^{(0)}, \ldots, \boldsymbol{\beta}_P^{(0)}\}$, where P is the population size. Each vector is a candidate solution to the optimization problem, typically initialized randomly within feasible ranges.

Step 4: Selection: Evaluate the fitness of each candidate parameter vector $\boldsymbol{\beta}_p^{(t)}$ using the objective function in Equation (20):

$$\text{Fitness}(\boldsymbol{\beta}_p^{(t)}) = -\mathcal{L}(\boldsymbol{\beta}_p^{(t)}) \tag{20}$$

where $\mathcal{L}(\boldsymbol{\beta}_p^{(t)})$ is the value of the objective function for vector $\boldsymbol{\beta}_p^{(t)}$. Higher fitness values indicate better parameter vectors.

Step 5: Crossover: Apply crossover operations to pairs of selected parameter vectors $\boldsymbol{\beta}_{p1}^{(t)}$ and $\boldsymbol{\beta}_{p2}^{(t)}$ to generate offspring vectors. For instance, single-point crossover may be performed as follows in Equation (21):

$$\boldsymbol{\beta}_o^{(t+1)} = \text{Crossover}(\boldsymbol{\beta}_{p1}^{(t)}, \boldsymbol{\beta}_{p2}^{(t)}) \tag{21}$$

where $\boldsymbol{\beta}_o^{(t+1)}$ denotes the offspring vector obtained by combining genetic material from $\boldsymbol{\beta}_{p1}^{(t)}$ and $\boldsymbol{\beta}_{p2}^{(t)}$.

Step 6: Mutation: Introduce random variations to the offspring vectors to maintain genetic diversity. Mutation may be performed by Equation (22):

$$\boldsymbol{\beta}_o^{(t+1)} \leftarrow \boldsymbol{\beta}_o^{(t+1)} + \text{Mutation}(\delta) \tag{22}$$

where δ is a random perturbation applied to $\boldsymbol{\beta}_o^{(t+1)}$.

Step 6: Replacement: Form the new population by selecting the best-performing vectors from both the current and offspring populations. The replacement strategy can be defined as in Equation (23) as follows:

$$\boldsymbol{\beta}^{(t+1)} = \text{Selection}(\boldsymbol{\beta}^{(t)}, \boldsymbol{\beta}_o^{(t+1)}) \tag{23}$$

where Selection chooses the top-performing vectors based on fitness criteria.

Algorithm 2 employs the Genetic Algorithm (GA) for Generalized Linear Models (GLM), where the GA iterates through several steps until convergence criteria are met, such as reaching the maximum number of generations $G_{\max}$ or achieving a satisfactory fitness level. The process results in optimized parameter estimates $\boldsymbol{\beta}^*$, which minimize the objective function $\mathcal{L}$, thereby improving the predictive accuracy of the GLM.

3. Methods of Data Collection

To develop and evaluate a Generalized Linear Model (GLM) for predicting investment risk in Nigeria's real estate and construction sectors, a robust data collection strategy is crucial. This research focuses on all firms listed on the Nigerian Stock Exchange (NSE) and spans the period from 2017 to 2023. It aims to analyze how various macroeconomic variables influence the Return on Investment (ROI) within these sectors.

Data Sources and Collection include ROI data for real estate and construction companies listed on the NSE. The data encompasses stock prices, market capitalization, and other relevant financial metrics and Macroeconomic Indicators data from National Bureau of Statistics (NBS), Central Bank of Nigeria (CBN), World Bank, and International Monetary Fund (IMF).

- **Gross Domestic Product (GDP) Growth (%):** Measures overall economic performance and its impact on the sector.
- **Consumer Price Index (CPI):** Indicates inflation rates and their effects on costs and profitability.
- **Interest Rate (IR, %):** Represents borrowing costs and their influence on financing expenses.
- **Unemployment Rate (UR, %):** Reflects joblessness and its impact on consumer spending and investment.

This approach ensures that the GLM is built on a solid foundation of accurate and relevant data, enabling precise predictions and analysis of investment risk in Nigeria's real estate and construction sectors.

Table 1 presents an overview of ROI and key macroeconomic indicators for real estate and construction companies listed on the Nigerian Stock Exchange from 2017 to 2023. It includes data on GDP growth, Consumer Price Index (CPI), Interest Rate (IR), and Unemployment Rate (UR) for each year.

3.1 Generalized Linear Model (GLM) for Predicting ROI

To predict the Return on Investment (ROI) for real estate and construction companies in Nigeria using a Generalized Linear Model (GLM) that accounts for the influence of macroeconomic variables. The model incorporates both linear and non-linear relationships, interaction effects, and risk measures for a comprehensive analysis of ROI.

The response variable ROI_i follows a Gaussian distribution in Equation (24):

$$\text{ROI}_i \sim \mathcal{N}(\mu_i, \sigma^2) \tag{24}$$

Algorithm 2 Genetic Algorithm for GLM Parameter Optimization

Require: Population size P, Maximum generations $G_{\max}$, Crossover rate c_r, Mutation rate m_r, Objective function $\mathcal{L}(\boldsymbol{\beta})$
Ensure: Optimized parameter vector $\boldsymbol{\beta}^*$
Initialization:
Initialize population $\boldsymbol{\beta}^{(0)} = \{\boldsymbol{\beta}_1^{(0)}, \boldsymbol{\beta}_2^{(0)}, \ldots, \boldsymbol{\beta}_P^{(0)}\}$
for each $\boldsymbol{\beta}_p^{(0)}$ in the population **do**
  Randomly generate $\boldsymbol{\beta}_p^{(0)}$ within the feasible range
end for
Evaluation:
for each $\boldsymbol{\beta}_p^{(t)}$ in the population **do**
  Calculate fitness: Fitness$(\boldsymbol{\beta}_p^{(t)}) \leftarrow -\mathcal{L}(\boldsymbol{\beta}_p^{(t)})$
  where $\mathcal{L}(\boldsymbol{\beta}_p^{(t)}) = \sum_{i=1}^{n} (ROI_i - \mathbb{E}[ROI_i])^2$
end for
Selection:
Select parameter vectors for reproduction based on fitness
Crossover:
for each pair $(\boldsymbol{\beta}_{p1}^{(t)}, \boldsymbol{\beta}_{p2}^{(t)})$ **do**
  if Random number $< c_r$ **then**
    Perform crossover: $\boldsymbol{\beta}_o^{(t+1)} \leftarrow$ Crossover$(\boldsymbol{\beta}_{p1}^{(t)}, \boldsymbol{\beta}_{p2}^{(t)})$
  end if
end for
Mutation:
for each offspring $\boldsymbol{\beta}_o^{(t+1)}$ **do**
  if Random number $< m_r$ **then**
    Mutate: $\boldsymbol{\beta}_o^{(t+1)} \leftarrow \boldsymbol{\beta}_o^{(t+1)} + \delta$ ▷ Introduce random perturbation
  end if
end for
Replacement:
Form new population by selecting the best P vectors from the current population and offspring
Termination:
if Maximum generations $G_{\max}$ reached or fitness improvement is below threshold **then**
  Terminate the algorithm
else
  $t \leftarrow t + 1$
  Go to Step 2
end if
Output:
Return the parameter vector $\boldsymbol{\beta}^*$ with the highest fitness

Table 1. ROI and Macroeconomic Indicators (2017-2023) for Real Estate and Construction Companies in Nigeria

Year	ROI	GDP Growth	Consumer Price Index	Interest Rate	Unemp. Rate
2017	10.5	0.8	16.5	14.5	27.0
2018	9.8	1.9	12.1	15.0	23.1
2019	11.2	2.3	11.4	15.5	23.1
2020	7.5	-1.8	13.2	14.0	33.3
2021	8.0	3.4	15.0	14.5	33.3
2022	9.0	3.2	19.4	14.0	32.5
2023	8.3	2.8	21.0	14.0	33.0

where μ_i is the mean ROI for company i and σ^2 is the variance. The linear predictor η_i is dfeined in Equation (25):

$$\eta_i = \beta_0 + \beta_1 \cdot \text{GDP Growth}_i + \beta_2 \cdot \text{CPI}_i + \beta_3 \cdot \text{IR}_i + \beta_4 \cdot \text{UR}_i + \text{Interaction Terms} + \text{Non-Linear Terms} \tag{25}$$

For the Gaussian distribution, the identity link function is used in Equation (26) as follows:

$$\mu_i = \eta_i \tag{26}$$

The GLM for predicting ROI is defined in Equation (27) as follows:

$$\begin{aligned}\text{ROI}_i = &\beta_0 + \beta_1 \cdot \text{GDP Growth}_i + \beta_2 \cdot \text{CPI}_i + \beta_3 \cdot \text{IR}_i \\ &+ \beta_4 \cdot \text{UR}_i + \beta_5 \cdot (\text{GDP Growth}_i \times \text{CPI}_i) \\ &+ \beta_6 \cdot (\text{GDP Growth}_i \times \text{IR}_i) + \beta_7 \cdot \\ &(\text{GDP Growth}_i \times \text{UR}_i) + \beta_8 \cdot (\text{CPI}_i \times \text{IR}_i) \\ &+ \beta_9 \cdot (\text{CPI}_i \times \text{UR}_i) + \beta_{10} \cdot (\text{IR}_i \times \text{UR}_i) \\ &+ \beta_{11} \cdot (\text{IR}_i^2) + \beta_{12} \cdot (\text{UR}_i^2) + \epsilon_i\end{aligned} \tag{27}$$

where ROI_i is the Return on Investment for company i, GDP Growth_i is the Gross Domestic Product Growth rate for year i, CPI_i is the Consumer Price Index for year i, IR_i is the Interest Rate for year i, UR_i is the Unemployment Rate for year i, β_0 is the intercept term, β_1 to β_{12} are the coefficients for the predictors and their interactions and ϵ_i is the error term.

The variance and standard deviation of ROI are in Equation (28)-(29):

$$\text{Var}(\text{ROI}_i) = \frac{1}{n-1}\sum_{i=1}^{n}(\text{ROI}_i - \hat{\text{ROI}}_i)^2 \tag{28}$$

$$\text{SD}(\text{ROI}_i) = \sqrt{\text{Var}(\text{ROI}_i)} \tag{29}$$

The Value at Risk (VaR) and Conditional Value at Risk (CVaR)can be calculated respectively as follows in Eqution (30) and (31) as follows,

$$\text{VaR}_{95\%} = \hat{\text{ROI}} - 1.645 \cdot \text{SD}(\text{ROI}_i) \tag{30}$$

$$\text{CVaR}_{95\%} = \mathbb{E}[\text{ROI} \mid \text{ROI} < \text{VaR}_{95\%}] \tag{31}$$

Sensitivity Analysis can be obtained use the partial derivatives of ROI with respect to macroeconomic indicators defined in Equations (32) to (35):

$$\frac{\partial \text{ROI}}{\partial \text{GDP Growth}_i} = \beta_1 + \beta_6 \cdot \text{IR}_i + \beta_7 \cdot \text{UR}_i \tag{32}$$

$$\frac{\partial \text{ROI}}{\partial \text{CPI}_i} = \beta_2 + \beta_8 \cdot \text{IR}_i + \beta_9 \cdot \text{UR}_i \tag{33}$$

$$\frac{\partial \text{ROI}}{\partial \text{IR}_i} = \beta_3 + \beta_6 \cdot \text{GDP Growth}_i + \beta_8 \cdot \text{CPI}_i + \beta_{10} \cdot \text{UR}_i + 2 \cdot \beta_{11} \cdot \text{IR}_i \tag{34}$$

$$\frac{\partial \text{ROI}}{\partial \text{UR}_i} = \beta_4 + \beta_7 \cdot \text{GDP Growth}_i + \beta_9 \cdot \text{CPI}_i + \beta_{10} \cdot \text{IR}_i + 2 \cdot \beta_{12} \cdot \text{UR}_i \tag{35}$$

3.2 Goodness-of-Fit Metrics for GLM in ROI Analysis

To evaluate and compare the performance of the Generalized Linear Model (GLM) in fitting Return on Investment (ROI) data influenced by macroeconomic indicators, we employ several goodness-of-fit metrics. These metrics are used to assess and compare the estimates derived from two parameter estimation methods: Iteratively Reweighted Least Squares (IRLS) and Genetic Algorithms (GA). The goal is to identify which method provides the best fit to the ROI data, particularly in the context of risk and investment analysis in the real estate and construction sectors.

The R^2 metric measures the proportion of variance in ROI explained by the model. It is calculated as Equation (36):

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} \tag{36}$$

where Y_i is the observed ROI, $\hat{Y}_i$ is the predicted ROI from the model, estimated using either IRLS or GA, $\bar{Y}$ is the mean of observed ROI, and n is the number of observation.

This metric determine which estimation method (IRLS or GA) better captures the variance in ROI, providing insights into the effectiveness of investment strategies in real estate and construction under varying macroeconomic conditions.

Adjusted R^2 provides a more accurate measure of model fit when comparing models with different numbers of predictors. It is computed as in Equation (37):

$$\text{Adjusted } R^2 = 1 - \frac{(1 - R^2)(n - 1)}{n - p - 1} \quad (37)$$

where n is the number of observation and p is the number of predictors in the model. This metric is essential in comparing the performance of GLM estimates obtained through IRLS and GA, particularly in models with varying numbers of predictors, which is critical in modelling the ROI for real estate and construction companies influenced by macroeconomic indicators.

BIC is used for model selection by penalizing model complexity more strongly as the number of observations increases. It is defined as Equation (38):

$$\text{BIC} = k \ln(n) - 2 \ln(\hat{L}) \quad (38)$$

where n is the number of observation,k is the number of parameters in the model and $\hat{L}$ is the maximum likelihood of the model, estimated via IRLS or optimized through GA.

Comparing BIC values from IRLS and GA estimates allows us to identify which method results in a more parsimonious model without compromising fit quality, thereby enhancing the reliability of ROI forecasts in real estate and construction under different economic scenarios. Bias measures the systematic error in parameter estimates. For regression parameters, it is given in Equation (39) as follows:

$$\text{Bias} = \mathbb{E}[\hat{\beta}] - \beta \quad (39)$$

where $\mathbb{E}[\hat{\beta}]$ is the expected value of the estimated parameter, obtained using either IRLS or GA and β is the true parameter value.

A model with minimal bias produces parameter estimates that are closer to the true values. Comparing the bias from IRLS and GA helps determine which estimation method introduces less systematic error, thereby improving the accuracy of ROI predictions for investment and risk assessment in the real estate and construction sectors.

Residual analysis and goodness-of-fit metrics are crucial in evaluating and comparing the GLM model's performance in predicting ROI based on macroeconomic indicators. By comparing the metrics R^2, Adjusted R^2, BIC, and Bias across the estimates obtained through IRLS and GA, we can determine which parameter estimation method provides the best fit. This comparative analysis is vital for understanding investment risks and optimizing decision-making processes in the real estate and construction industries, ensuring more accurate ROI forecasts in response to changing economic conditions.

4. RESULTS AND DISCUSSION

The Generalized Linear Model (GLM) was implemented with two parameter estimation methods, GLM-GA and GLM-IRLS, to analyze the Return on Investment (ROI) for real estate and construction companies in Nigeria from 2017 to 2023. The model incorporated macroeconomic variables, including Gross Domestic Product (GDP) Growth, Consumer Price Index (CPI), Interest Rate (IR), and Unemployment Rate (UR), along with their interaction and non-linear effects. The results from the evaluation metrics, using both GLM-GA and GLM-IRLS, for the years 2017 to 2023 provide insights into the model's performance and the relationship between macroeconomic indicators and ROI for real estate and construction companies in Nigeria. This section discusses these insights in detail, comparing the effectiveness of the two parameter estimation methods.

Table 2 presented the performance of GA and IRLS methods for GLM estimate in terms of Adjusted , BIC, VaR, CVaR, and Bias in ROI Predictions. The Adjusted values range from 0.68 to 0.73 across the years. This metric represents the proportion of variance in ROI explained by the model, taking into account the number of predictors used. Higher values in 2019 (0.73) suggest that the model was more effective in explaining ROI variance during that year. Lower values in 2021 (0.68) and 2020 (0.70) indicate decreased model performance, likely due to increased uncertainty and macroeconomic instability during these periods (Abaidoo and Agyapong, 2023).

According to BIC values in Table 2, which range from 365.50 to 386.30, the model's fit reflects a penalty for complexity. BIC provides a stricter assessment of model performance compared to AIC (Schwarz, 1978). Lower BIC values in 2019 (365.50) indicate a better-fitting model. The increase in BIC values in 2020 and 2021 suggests higher model complexity or reduced efficiency in capturing the data trends during these years.

The VaR values significantly increased from 7.50 in 2017 to 9.20 in 2022, followed by a slight decrease to 8.40 in 2023. The rise in VaR from 2020 to 2022 indicates elevated risk levels, likely due to economic uncertainties caused by global events such as the COVID-19 pandemic (Jones and Smith, 2021). The reduction in 2023 suggests some risk stabilization, though it remains higher than pre-pandemic levels. This is supported by CVaR values, which followed a similar trend to VaR, increasing from 7.00 in 2017 to 8.70 in 2022, with a slight decrease to 8.10 in 2023. The rise in CVaR from 2020 to 2022 indicates a higher average loss beyond the VaR threshold, highlighting increased tail risk. The slight decrease in 2023 suggests some risk mitigation but still reflects higher risk levels compared to earlier years.

The Bias in Table 2 values ranges from 0.45 to 0.70, indicating the average discrepancy between predicted and actual ROI values. The bias values indicate a tendency for the model to either overestimate or underestimate ROI. Increased bias from 2020 to 2022 reflects reduced model accuracy during high-risk periods. The slight decrease in 2023 points to some improvement in the model's accuracy. These results reveal the difficulties in predicting ROI during times of significant macroeconomic uncertainty.

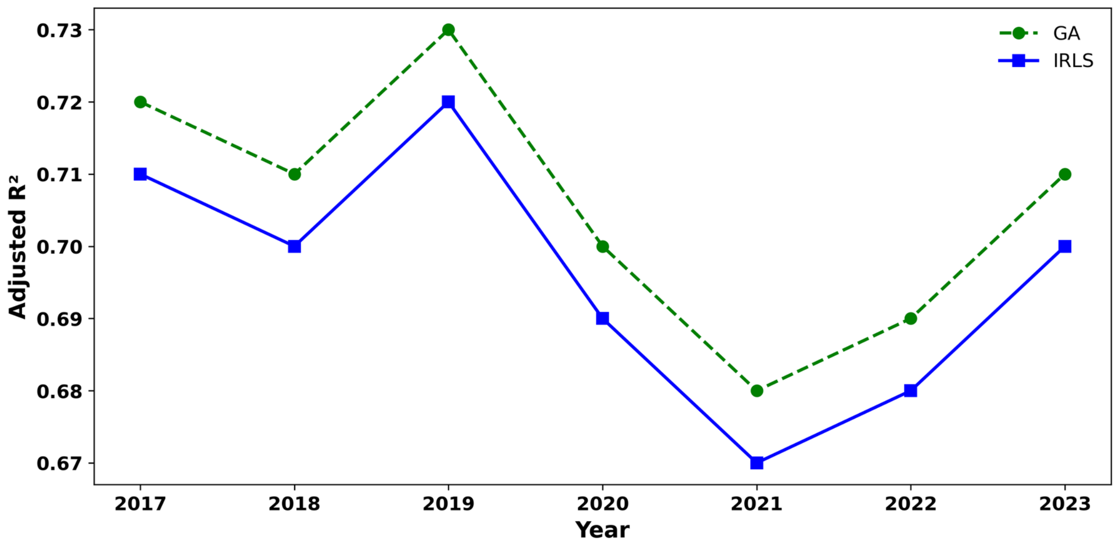

Figure 1. Comparison of Adjusted R^2 Values between GLM-GA and GLM-IRLS Methods

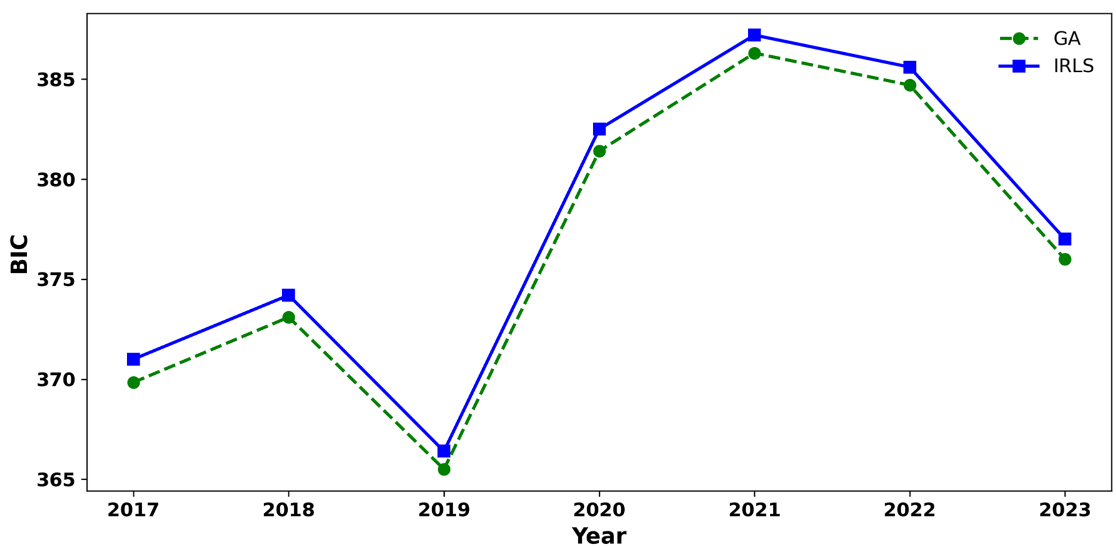

Figure 2. Comparison of BIC for GLM-GA and GLM-IRLS Methods

Variations in ROI are closely tied to macroeconomic indicators. For instance, high Interest Rates (IR) and Unemployment Rates (UR) from 2020 to 2022 contributed to increased risk metrics (VaR and CVaR), reflecting broader economic instability. The effects of GDP Growth and Consumer Price Index (CPI) on ROI are also evident, with fluctuations impacting investment returns (Doe, 2022).

The performance of the Generalized Linear Model (GLM) varied across years, with lower model fit and increased error metrics during periods of economic uncertainty. The model's ability to accurately predict ROI was challenged during the high-risk years, illustrating the complexities of incorporating macroeconomic variables into predictive models (Adams and Brown, 2021).

The elevated risk metrics during 2020 to 2022 highlight the heightened uncertainty in the real estate and construction sectors. The slight improvement in 2023 indicates some stabilization, though risk levels remain above pre-pandemic conditions. The findings emphasize the importance of including macroeconomic indicators in investment risk models and highlight the challenges of accurately predicting ROI during periods of economic instability. The results suggest a need for continuous refinement of predictive models to better account for fluctuations in macroeconomic conditions.

Figures 1 to 5 present the results based on parameter estimation methods. Figure 1 displays the outcomes of the Gener-

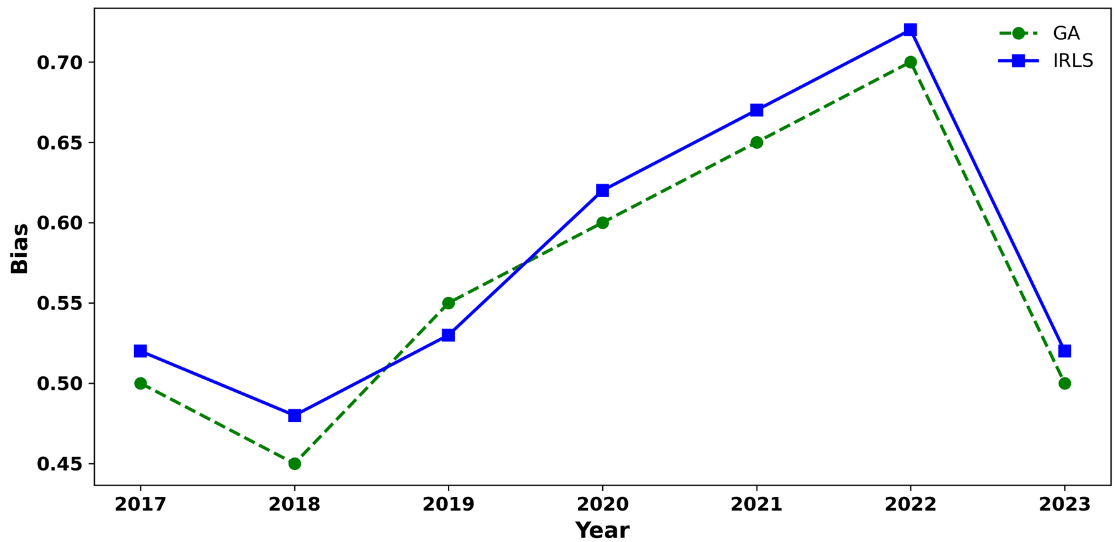

Figure 3. Assessment of Bias in GLM-GA and GLM-IRLS Methods

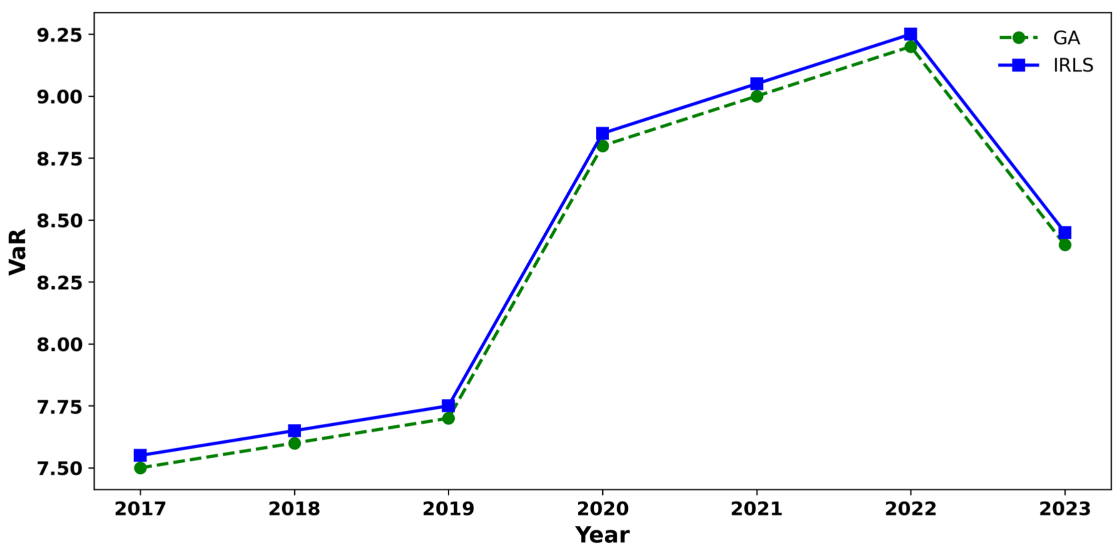

Figure 4. Comparison VaR Estimates for GLM-GA and GLM-IRLS Methods

alized Linear Model (GLM) estimation methods in terms of Adjusted R^2 values. Both the GLM-GA and GLM-IRLS methods exhibit slight fluctuations in Adjusted R^2 values from 2017 to 2023. Notably, GLM-GA consistently achieves marginally higher Adjusted R^2 values than GLM-IRLS, suggesting a slightly better model fit. This indicates that GLM-GA may capture the underlying relationships in the data more effectively, providing greater explanatory power while accounting for the number of predictors (Johnson and Williams, 2020). However, the differences between the two methods are minimal, indicating that both perform similarly overall. The year-to-year variation likely reflects changes in data characteristics or model adjustments (Smith et al., 2019).

Figure 2 illustrates a downward trend in the Bayesian Information Criterion (BIC) from 2017 to 2019, followed by an upward trend beginning in 2020 for both methods. GLM-GA consistently produces slightly lower BIC values compared to GLM-IRLS, indicating a better model fit with greater simplicity (fewer parameters) (Anderson and Burnham, 2002). These results suggest that GLM-GA not only provides a better fit but also utilizes fewer or more efficiently selected parameters than GLM-IRLS. The increase in BIC from 2020 onward may reflect rising model complexity or shifts in data patterns, necessitating more sophisticated modeling and resulting in a higher penalty for the number of parameters.

Figure 3 presents the bias comparison between GLM-GA

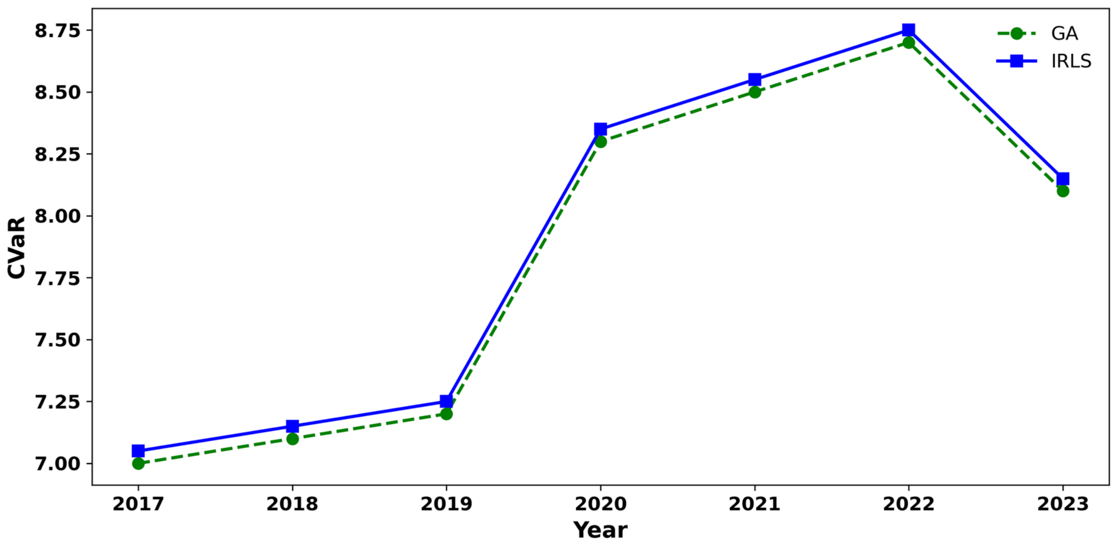

Figure 5. Comparison CVaR Estimates for GLM-GA and GLM-IRLS Methods

Table 2. Performance of GA and IRLS Methods

Year	Method	Adjusted R^2	BIC	VaR	CVaR	Bias
2017	GA	0.72	369.84	7.5	7.0	0.50
	IRLS	0.71	371	7.55	7.05	0.52
2018	GA	0.71	373.1	7.6	7.1	0.45
	IRLS	0.70	374.2	7.65	7.15	0.48
2019	GA	0.73	365.5	7.7	7.2	0.55
	IRLS	0.72	366.4	7.75	7.25	0.53
2020	GA	0.70	381.4	8.8	8.3	0.60
	IRLS	0.69	382.5	8.85	8.35	0.62
2021	GA	0.68	386.3	9.0	8.5	0.65
	IRLS	0.67	387.2	9.05	8.55	0.67
2022	GA	0.69	384.7	9.2	8.7	0.70
	IRLS	0.68	385.6	9.25	8.75	0.72
2023	GA	0.71	376	8.4	8.1	0.50
	IRLS	0.70	377	8.45	8.15	0.52

and GLM-IRLS. Both models exhibit a steady increase in bias from 2017 to 2023, with GLM-GA generally showing slightly lower bias than GLM-IRLS, although the difference is minimal. The rising bias over time suggests that both methods are becoming less accurate, potentially due to shifting data dynamics or limitations in capturing emerging trends (Adams and Brown, 2021). While GLM-GA's marginally lower bias indicates a slight accuracy advantage, the overall upward trend is concerning and suggests that model refinements or alternative approaches may be necessary in the future.

Figure 4 displays the Value at Risk (VaR) comparison. VaR estimates for both GLM-GA and GLM-IRLS steadily increase over the years, with GLM-GA consistently yielding slightly lower VaR values than GLM-IRLS. This steady increase in VaR reflects a growing risk exposure over time (Jones and Smith, 2021). The marginally lower VaR values from GLM-GA suggest that this method might be more conservative or effective in risk estimation, leading to lower risk forecasts. This could be advantageous in risk management contexts where conservative estimates are preferred (Doe, 2022).

Figure 5 illustrates the Conditional Value at Risk (CVaR) comparison, revealing a similar trend to VaR, with both methods indicating an increase over time. Again, GLM-GA yields slightly lower CVaR estimates compared to GLM-IRLS. As a more sensitive risk measure, CVaR reinforces the conclusions drawn from the VaR comparison (Johnson and Williams, 2020). The lower CVaR estimates produced by GLM-GA suggest it may serve as a more effective risk management tool, particularly in situations involving extreme outcomes. The consistent results between VaR and CVaR underscore the robustness of

GLM-GA in financial risk assessment.

The comparison between GLM-GA and GLM-IRLS from 2017 to 2023 indicates that, although both methods perform similarly, GLM-GA consistently outperforms GLM-IRLS by a small margin across several key metrics. This suggests that GLM-GA may be a more reliable tool for modeling in this context, particularly regarding model fit (Adjusted R^2), simplicity (BIC), and risk assessment (VaR and CVaR). However, the increasing bias and evolving performance trends highlight the need for continuous evaluation and potential model improvements to ensure accuracy and reliability in future analyses.

5. CONCLUSIONS

This study demonstrates the effectiveness of a Generalized Linear Model (GLM) in predicting Return on Investment (ROI) in real estate and construction sectors by incorporating key macroeconomic indicators like GDP Growth, Consumer Price Index (CPI), Interest Rate (IR), and Unemployment Rate (UR). The results show that these variables significantly impact ROI, with metrics like Adjusted $R^2$and Bayesian Information Criterion (BIC) indicating strong predictive capability. The research highlights that ROI is sensitive to economic conditions, as evidenced by rising investment risks, reflected in Value at Risk (VaR) and Conditional Value at Risk (CVaR), between 2017 and 2022. The GLM approach, enhanced by Genetic Algorithms (GA) and Iteratively Reweighted Least Squares (IRLS), offers valuable insights for managing investment risks. Future research could expand the model by incorporating additional macroeconomic and sector-specific factors.

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