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Research Paper

Dynamic Modeling of Energy Data: World Crude Oil and Coal Prices 2017-2023 (A State-Space Model Analysis of Multivariate Time Series)

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Abstract
The analysis of global crude oil and coal prices has attracted considerable research interest, as these prices significantly affect both society and industry, making the topic highly relevant for governments and policy makers. This study examines the correlation between global coal and crude oil prices from 2017 to 2023. It analyzes the behavior of these price series using a unit root test and develops an optimal model for conducting a Granger-causality analysis. To forecast crude oil and coal prices for the next 30 periods, a state-space modeling approach is applied. The unit root test results reveal that these prices are non-stationary, suggesting that any shocks to prices will have persistent effects. The best-fitting model for the association between coal and crude oil prices is a vector autoregressive model of order two (VAR(2)). The Granger-causality results reveal that current crude oil prices are influenced by both their own past values and previous coal prices, and vice versa. Forecasts using the state-space model suggest a modest upward trend for crude oil prices over the next 30 periods, while coal prices are projected to rise more strongly.

Keywords
Unit Root Test, VAR(p) Model, Granger-Causality, State Space Model, Forecasting

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1. INTRODUCTION

Multivariate time series analysis examines several time series data, and is a subfield of multivariate statistics that focuses on analyzing interdependent data (Hamilton, 1994; Lutkepohl, 2005; Harvey, 2009; Tsay, 2010; Tsay, 2014; Wei, 2006; Wei, 2006; Basu et al., 2019). Multivariate time series are generally more complex than univariate time series, particularly when working with a large amount of data.

One technique for examining sequential data acquired over time is time series analysis which is important because it is an attempt to accurately comprehend (estimate) past, present, and future values for occurrences of relevance to researchers. Using these results, researchers can learn about events that are interesting, their structure and influence, or countermeasures. The techniques for precisely estimating past, present, and future values are the main focus of this time series analysis (Hagiwara, 2021). To estimate future event, by using historical data and information, time series analysis is applied (Lutkepohl, 2005; Tsay, 2014; Wei, 2019; Hagiwara, 2021).

Time series analysis specifically accounts for the relationships between observations across different time points. Typically, time series data reveal connections between values recorded at one moment and those at another (Lutkepohl, 2005; Harvey, 2009; Tsay, 2014; Wei, 2019). Sims (1980) was a pioneer who introduced Vector Autoregressive (VAR) model as an another approach for analyzing macroeconomic data. The VAR model is often applied to analyze variables that mutually influence one another, assuming the data is stationary. In multivariate time series modeling, the focus is on the interdependence between variables based on lagged values (Wei, 2006; Wei, 2019; Tsay, 2014; Kilian and Lutkepohl, 2019; Usman et al., 2022; Russel et al., 2023). The goal of multivariate time series analysis is to create a model that captures the time-based interactions between several variables, which can then be used for tasks such as forecasting (Florens et al., 2007). Within the VAR framework, the lagged values of variables serve as the independent variables, resulting in a simpler model with fewer parameters, where all variables are treated as endogenous (Tsay, 2014; Wei, 2019; Hagiwara, 2021).

Stationary variables without trends are the focus of the VAR model. The stochastic characteristics of the data indicate that multivariate time series often include both long-term and short-term components. The subject of how to establish potential linkages within multivariate time series data. What are some potential strategies for modeling two or more multivariate time series data? When every variable in the structure is interdependent, the univariate modeling approach is not applicable anymore.

The VAR (Vector Autoregression) model focuses on stationary variables that do not exhibit trends. The stochastic characteristics of the data indicate that multivariate time series often include both long-term and short-term elements. This raises the question of how to establish potential relationships within multivariate time series data. What methods can be used to model two or more multivariate time series variables. When all variables within the system are interrelated, using a univariate modeling approach is no longer appropriate.

In this research, the discussion of data analysis will use stochastic methods based on state-space models as a type of more in-depth analysis in an attempt to find accurate forecasting values. The optimal VAR model will serve as the foundation for the state space model, which chosen according to the best model selection conditions, such as AIC (Akaike Information Criterion). The canonical correlation criterion will be used to establish the most appropriate state vector for the modeling process based on the chosen VAR model (Wei, 2006). The state space modeling framework was initially introduced by Kalman and Bucy (1961) and Kalman (1960). This approach is broadly applied.

The state space model was employed successfully, especially in the military applications. State-space models were used in statistical analysis relatively recently, around the seventies, scientists in the field of statistics discovered that this approach could be used to time series analysis in general (Akaike, 1973; Akaike, 1974). Since that time, the state-space method has been used in various research areas, including in the research fields: political science, finance, business, economics, biology, environmental science, and even medical science Siontis et al., 2021. Hannan and Deistler (1988) offered a unified treatment of state-space modeling, bringing together contributions from time series engineers and analysts focused on stationary time series. Aoki (1994) explored multivariate state-space models within economics and introduced techniques applicable to both stationary and nonstationary datasets. The use of state-space methods has also been widely used in machine learning (Gu et al., 2021; Gu et al., 2022, Gu and Dao, 2023, Lin and Michailidis, 2024, Rangapuram et al., 2018, Sanchez-Bornot and Sotero, 2023).

In this study, its will discuss how energy data behaves, namely world oil and coal prices from 2017 to 2023. The VAR and state-space models are used in order to obtain exact forecasting. Next, the Granger Causality will be used to see the causal connection between variables. The data consists of world coal and crude prices data from 2017 to 2023. This research objectives are to: (1) find the best VAR(p) model for the data, (2) formulate a state-space model; (3) use Granger Causality test to examine the dynamic behavior of data; and (4) make forecasts for several future data periods.

Statistical analysis using state-space models has developed rapidly (Hamilton, 1994; Wei, 2006; Harvey, 2009; Prado and West, 2010; Durbin and Koopman, 2012; Gomez, 2016; Hagiwara, 2021). Some characteristics of state-space models: first, the introduction of latent variables, namely states, allows this model to be applied as a foundation for simply building complex models by combining many states; Furthermore, represents relationships between observations through states indirectly and not through direct relationships between observations (Durbin and Koopman, 2012; Gomez, 2016; Hagiwara, 2021). Over the past few decades, its application has grown in a variety of sectors. Pandher (2002) discusses macroeconomics and finance using the state-space model; Heiss (2008) discusses panel data macroeconomics with state-space models; Pao (2009) discusses electricity use and economic growth using a state-space time series model; Weiqi et al. (2012)) conducted research on crude oil prices forecasting using the state-space model; Ramos et al. (2015) conducted research for forecasting retail sales using a state-space model; Snyder et al. (2017) conducted research on forecasting market share in car sales from 1961-2013 in the US; Smith and Maneesoonthorn (2018) conducting research into forecasting inflation and electricity inflation using the state-space method; Hu et al. (2019) conducted research on forecasting electricity demand using the state-space method; Kurniati et al. (2019) and Nagbe et al. (2018) conducted forecasting research on electricity demand using the state-space method; Russel et al. (2023) discussed multivariate time series data on inflation for several economic sectors in Indonesia using the state-space method.

Hamilton (1994), West and Harrison (1997), Durbin and Koopman (2012), Harvey (2009), and Gomez (2016) provide a review of state-space models, while Wei (2006) defines it as a method for concurrently modeling and forecasting a number of interrelated time series data variables in which the variables interact dynamically. The state-space method's primary objective is to use the collected observations to deduce pertinent outcomes from a set of vectors. Because of its versatility, the state-space approach is able to be used to both univariate and multivariate data. For an extensive information about state-space applications in multivariate time series, see (Gomez, 2016). Theoretical books discussing this model can be seen in (Hamilton, 1994; Wei, 2006; Commandeur and Koopman, 2007; Harvey, 2009; Durbin and Koopman, 2012; Gomez, 2016; Hagiwara, 2021). This approach does not necessitate a connection between the observed variables, and is often used on data containing a single variable. To determine the link between sets of observed variables, this paper discusses the use of the state-space technique on multivariate data and its representation in VAR. The relationship between variables in the data can be explained analytically using the VAR model.

Diebold (1989) explains that According to Professor Aoki's foreword, the goal of state-space modeling of time series is to bridge the gap between engineering and econometric research. This is built on the work of Aoki (1987) and provides econometricians with a valuable tool for empirical applications. Diebold (1989) argues the superiority of this model when used on multivariate data by producing more accurate predictions. To create the model, Akaike (1973) and Akaike (1974) established the state vector as a vector of canonical variables linking data and future observations. Wei (2006) talks about fitting the state-space model via canonical correlation, and this design is able to be considered as an extension of a related concept for two vectors in a factor model. Because dynamic factors cannot be a combination of past and unseen future realizations of data vectors, unlike static factor models, this is a true generalization. Wei (2006) talks about fitting state-space models using canonical correlation.

According to Durbin and Koopman (2012), the state-space model offers a valuable foundation for real-world time series research in a number of domains, such as engineering, econometrics, business, and finance. One model that is able to be applied to analyze multivariate time series data is the state-space model, which was first presented by Kalman (1960). Multivariate time series with dynamic interactions are typically forecasted and modeled using the state-space approach. Compared to methods that represent each variable independently, state-space models can yield more accurate estimates by taking into account autocorrelation between all variables. This model provides a standard technique to analyse many difficulties in time series data (Durbin and Koopman, 2012).

2. EXPERIMENTAL SECTION

2.1 Methods

In multivariate time series modeling, several assumptions regarding the data must be checked, namely whether the variables to be analyzed meet the assumptions for the modeling that will be used. One of the important assumptions that must be met for VAR modeling is the stationarity assumption. Besides that, the stationarity and non-stationary nature of data have interesting implications for analysis (Parreno, 2024; Sari et al., 2024). Therefore, before the model is built, it must be checked for stationarity and cross-correlation among the variables (Hamilton, 1994; Brockwell and Davis, 1991; Wei, 2006; Madsen, 2008; Tsay, 2010; Warsono et al., 2019a; Warsono et al., 2019b; Warsono et al., 2020; Russel et al., 2023; Sari et al., 2024). The data plot behavior can be used to verify the data stationarity, and the ADF (Augmented Dickey-Fuller) test can be used to assess the data (Hamilton, 1994; Wei, 2006; Wei, 2019; Tsay, 2010). To confirm that there is a connection among the variables, the t-test is used and the cross correlation graphic representation is displayed (Pena et al., 2001; Wei, 2006; Wei, 2019; Tsay, 2010; Tsay, 2014). The Equation 1 model administers the Augmented Dickey-Fuller (ADF-test):

$$\Delta X = c + \alpha_t + \beta X_{t-1} + \sum_{i=1}^{m} \gamma_i \, \Delta X_{t-1} + e_t \tag{1}$$

The following are the alternative hypotheses and the null hypothesis (H_0):

$$H_0 : \beta = 0$$

against

$$H_1 : \beta < 0$$

The test-τ, commonly referred to as the Dickey-Fuller test, is a statistical procedure used to test H_0 as outlined in Equation 2.

$$\tau = \frac{\beta}{S_\beta} \tag{2}$$

If the p-value for $\alpha = 0.05$ is less than or equal to α, then reject H_0 (Wei, 2006; Enders, 2015; Virginia et al., 2018; Warsono et al., 2019a; Warsono et al., 2019b).

2.1.1 Vector Autoregressive (VAR) Model

The k-dimensional multivariate time series process is stationary process if each of its component series is univariate stationary and its first two moments are time invariant (Wei, 2019). According to Hamilton (1994), Pena et al. (2001), Wei (2006), Wei (2019), and Tsay (2014), the VAR model is able to be applied to analyze stationary data. The k-dimensional VAR model of order p, VAR(p) model, is given in Equation 3:

$$X_t = \gamma_0 + \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \ldots + \Phi_p X_{t-p} + \varepsilon_t \tag{3}$$

X_t is a $k \times 1$ vector time series, γ_0 is a $k \times 1$ vector constant, Φ_i is a $k \times k$ matrix of parameters (for $i > 0$, $\Phi_p \neq 0$), ε_t is vector shock with mean vector zero, and ε_t variance covariance matrix is Σ_ε. Relatively, the VAR(p) model is easy to be estimated. The maximum likelihood (ML), Bayesian, and Least Squares (LS) methods can all be used (Tsay, 2014; Wei, 2019).

2.1.2 Granger Causality Test

Determining if there are any causal effects among the variables being addressed is one of the intriguing challenges in analyzing a multivariate time series. The Granger causality test is designed to examine the short-term reciprocal link between the variables under investigation (Hamilton, 1994; Lutkepohl, 2005; Wei, 2006; Wei, 2019; Warsono et al., 2020; Russel et al., 2023). For instance, the Granger causality between variables Y and X is analyzed, and the Granger Causality Test model as shown in Equation 4:

$$\begin{aligned} Y_t &= c_1 + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \cdots + \gamma_p Y_{t-p} + \beta_1 X_{t-1} \\ &\quad + \beta_2 X_{t-2} + \cdots + \beta_p X_{t-p} + a_t \end{aligned} \tag{4}$$

The null hypothesis to be tested using ordinary least squares (OLS) is the following:

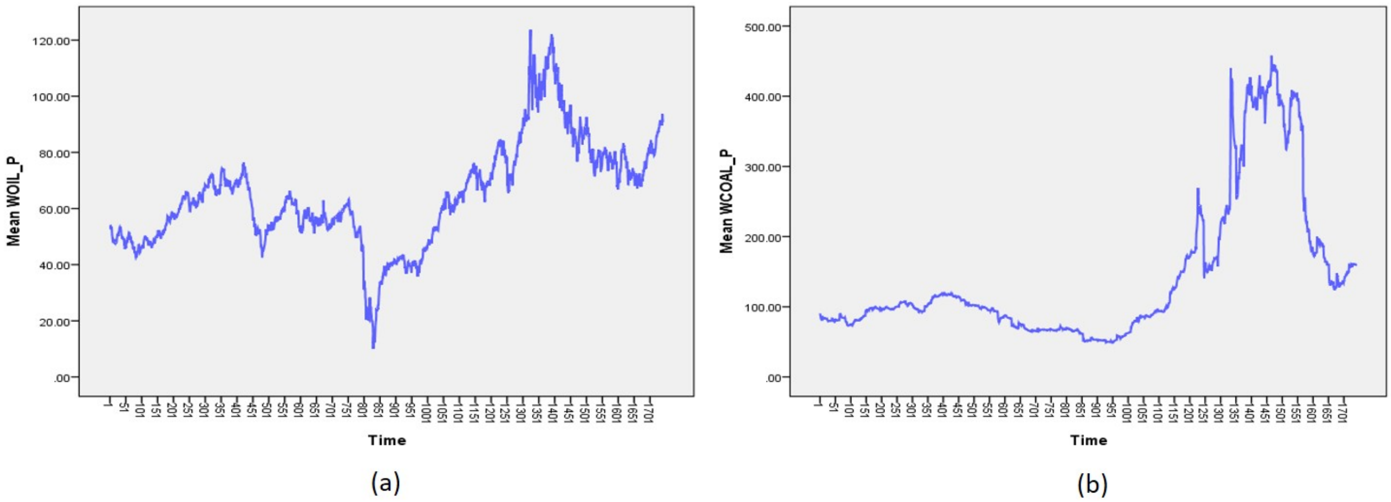

Figure 1. (a) World Crude Oil Prices from 2017 to 2023, (b) World Coal Prices from 2017 to 2023

$H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0$ (X is not Granger Causal Y), against H_1: at least one of $\beta_i \neq 0, i = 1, 2, \ldots, p$ (X Granger Causal Y), and Equation 5 is the test.

$$F \text{ Test} = \frac{(RSS_0 - RSS_1)/p}{RSS_1/(T - 2p - 1)} \tag{5}$$

According to Hamilton (1994), reject H_0 if p-value < 0.05. The residual sum of squares 1 (RSS_1) can be calculated using the shocks of Equation 5 as shown in Equation 6.

$$RSS_1 = \sum_{t=1}^{T} \hat{a}_t^2 \tag{6}$$

Equation 4 can be stated as Equation 7 under the null hypothesis:

$$Y_t = c_0 + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \cdots + \gamma_p Y_{t-p} + u_t \tag{7}$$

The shocks of Equation 7 can be used to determine the residual sum of squares, or RSS0, as shown in Equation 8 as follows:

$$RSS_0 = \sum_{t=1}^{T} \hat{u}_t^2 \tag{8}$$

2.1.3 State-Space Model

The state-space model from a system is a fundamental concepts of control theory and has associated Kalman filter recursions and have great impact on the analysis time series data (Brockwell and Davis, 1991; Wei, 2006; Hannan and Deistler, 1988) a process's state is defined by calculating the bare minimum of evidence from the past and present, so that the information from the present state and the future input can fully predict the behavior of the process in the future. Auxiliary variables, also known as state vectors, which some may not be directly noticeable, are used by the State-Space Model (SSM) to represent a multivariate time series (SAS/ETS 13.2, 2014). When predicting future values of the time series, the state vector compiles all of the evidence from the past and present values of the series. There are numerous SSM forms. The Akaike (1976) approach is the basis for the SSM form that is utilized here. The Equation 9 is the equation for state transition defines the model.

$$Y_{t+1} = FY_t + Ge_{t+1} \tag{9}$$

The output equation is given in Equation 10.

$$X_t = HY_t \tag{10}$$

where Y_t is an $s \times 1$ state vector, F is a transition matrix $s \times s$, G is an $s \times k$ input matrix, e_{t+1} is an $n \times 1$ vector of innovation, and H is the $m \times k$ output or observation matrix (Wei, 2006).

2.1.4 Canonical Correlation Analysis

A statistical technique called canonical correlation analysis is applied to determine the link between a set of independent variables and a set of dependent variables. The observations set of current and past events and a set of observations of current and future events can be analyzed to derive the state vector in a unique way, according to Akaike (1973), Akaike (1974), Akaike (1976), and Wei (2006). Wei (2006) discusses the canonical correlation in detail. The process of doing a canonical correlation analysis between data spaces is straightforward as shown in Equation 11.

Table 1. Dickey-Fuller Unit Root Tests Before and After Differencing First (d=1)

Variable	Type	Before Differencing			After Differencing First (d=1)			
		Rho	p-value	Tau	Rho	p-value	Tau	p-value
WOIL_P	Zero Mean	0.04	0.6917	0.04	0.6946	−1949.60	−31.19	<0.0001
	Mean	−6.35	0.3203	−1.67	0.4448	−1950.60	−31.19	<0.0001
	Single Mean	−10.65	0.3940	−2.30	0.4305	−1951.20	−31.18	<0.0001
	Trend	−8.62	0.5054	−0.55	0.4777	−1468.30	−27.08	<0.0001
WCOAL_P	Zero Mean	−0.82	0.6215	−1.31	0.6292	−1468.40	−27.08	<0.0001
	Mean	−3.29	0.6215	−1.31	0.6292	−1468.40	−27.08	<0.0001
	Single Mean	−4.61	0.8480	−1.45	0.8447	−1468.60	−27.07	<0.0001
	Trend	−4.61	0.8480	−1.45	0.8447	−1468.60	−27.07	<0.0001

Table 2. Representation of Cross-Correlation Schematically

Variable/Lag	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
WOIL_P	++	.	+	.	-	.	+	.	.	.	.	+	.	.	+	.	.	+	.	.	-
		++	.	.	.	.	.	.	+	.	.	.	.	.	.	.	.	.	.	.	.
WCOAL_P	++	.	++	.	-	.	+	.	.	.	.	.	.	.	.	.	++	.	.	.	.
			+	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Table 3. Minimum Information Criterion Based on AIC

Lag	AR0	AR1	AR2	AR3	AR4	AR5
AIC	4.7656	4.7699	4.7515	4.7538	4.7522	4.7569

$$D_n = \left(X'_n,\ X'_{n-1},\ \ldots,\ X'_{n-p}\right)' \tag{11}$$

and predictor space in Equation 12:

$$F_n = \left(X'_n,\ X'_{n+1|n},\ \ldots,\ X'_{n+p|n}\right)' \tag{12}$$

The least AIC (Akaike's Information Criterion) value determines the ideal fit of the data to the VAR(p) model, yielding the order p. The Block Hankel matrix representing the covariance between $D_n = (X'_n, X'_{n-1},..., X'_{n-p})'$ and $F_n = (X'_n, X'_{n+1|n}, ..., X'_{n+p|n})'$ is referred to as the canonical correlation analysis as shown in Equation 13.

$$\hat{\Gamma} = \begin{bmatrix} \hat{\Gamma}(0) & \hat{\Gamma}(1) & \cdots & \hat{\Gamma}(p) \\ \hat{\Gamma}(1) & \hat{\Gamma}(2) & \cdots & \hat{\Gamma}(p+1) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\Gamma}(p) & \hat{\Gamma}(p+1) & \cdots & \hat{\Gamma}(2p) \end{bmatrix} \tag{13}$$

where $\hat{\Gamma}(j)$ is the covariance matrix sample defined as in Equation 14.

$$\hat{\Gamma}(s) = \frac{1}{n}\sum_{t=1}^{n-s}(X_t - \bar{X})(X_{t+s} - \bar{X})' \tag{14}$$

The steps in building the state vector model in this study is using the canonical correlation approach follows the procedures given by Akaike (1976) and Wei (2006).

2.1.5 Forecasting

Predicting or estimating the future values is an important part of analysis in time series data analysis (Tsay, 2014; Kurniasari et al., 2025). The Kalman filter is a widely used method for approximating and estimating statistics in state-space models. Here, it is capable of adapting to changes in variances and model parameters. As noted by Welch et al. (2001), during the forecasting process, the current state estimate is produced, and the associated error covariance serves as the initial guess for the subsequent procedure. The Kalman filter operates as a recursive updating procedure, beginning with an initial state estimate and then refining that estimate by applying a corrective adjustment. Averages and covariance matrices are updated using the fundamental recursive formula (Wei, 2006). Using Equations 9 and 10, the Equations 15 and 16 can be utilized to compute the l-step advance forecasts from the forecast origin time t (Wei, 2006):

Table 4. Model Parameter Estimates and Testing for VAR(2) Model

Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
WOIL_P	CONST1	0.02185	0.03999	0.55	0.5849	1
	AR1_1_1	0.01565	0.02403	0.65	0.5149	WOIL_P (t-1)
	AR1_1_2	-0.00576	0.00619	-0.93	0.3520	WCOAL_P (t-1)
	AR2_1_1	-0.07356	0.02403	-3.06	0.0022	WOIL_P (t-2)
	AR2_1_2	0.02431	0.00619	3.93	0.0001	WCOAL_P (t-2)
WCOAL_P	CONST2	0.03589	0.15530	0.23	0.8173	1
	AR1_2_1	0.00249	0.09330	0.03	0.9787	WOIL_P (t-1)
	AR1_2_2	-0.02685	0.02403	-1.12	0.2641	WCOAL_P (t-1)
	AR2_2_1	0.18365	0.09332	1.97	0.0492	WOIL_P (t-2)
	AR2_2_2	0.09340	0.02404	3.89	0.0001	WCOAL_P (t-2)

Table 5. Test Granger-Causality

Test	Group Variable	Ho	Chi-Squares	*p*-value	Granger Cause
1	Group 1 Variable: WOIL_P	World crude oil price (WOIL_P) is affected by itself and unaffected by past information of world coal price (WCOAL_P).	16.82	0.0008	Significant
	Group 2 Variable: WCOAL_P				
2	Group 1 Variable: WCOAL_P	World coal price (WCOAL_P) is affected by itself and unaffected by past information of world crude oil price (WOIL_P).	14.89	0.0019	Significant
	Group 2 Variable: WOIL_P				

$$
\begin{aligned}
&= F\,\hat{Y}_t(l-1)\\
&= F\cdot F\,\hat{Y}_t(l-2)\\
&\;\vdots\\
&= F^l\,\hat{Y}_t
\end{aligned}
\tag{15}
$$

Hence,

$$
\begin{aligned}
\hat{X}_t(l) &= E\left(X_{t+l}\,\middle|\,X_j,\ j \le t\right)\\
&= H\,\hat{Y}_t(l)\\
&= H\,F^l\,\hat{Y}_t.
\end{aligned}
$$

where

$$\hat{Y}_t = E\left(Y_t\,\middle|\,Y_j,\ j \le t\right) = Y_t. \tag{16}$$

Clearly, from 15, the accuracy of the forecasts $\hat{X}_t(l)$ based on the estimate's quality of $\hat{Y}_t$ of the state vector Y_t, which compiles the data from the past required to make future predictions. The state vector must be updated whenever a new observation becomes available, and consequently, the forecast also must be updated, in order to enhance forecasts. The Kalman Filter method, a recursive process for drawing conclusions about the state vector Y_t, is accessible for this purpose (Wei, 2006; Gomez, 2016).

3. RESULTS AND DISCUSSION

World crude oil prices and world coal prices from 2017 to 2023 are discussed in this study. Data used can be obtained from website: `https://tradingeconomics.com/commodity/coal` and `https://tradingeconomics.com/commodity/crude-oil`. Figure 1(a) shows that crude oil prices from 2017 to 2018 appear to have an upward trend with relatively high price fluctuations. At the beginning of 2019 there appeared to be a downward trend and then a flat (constant) trend, but with relatively high fluctuations. In 2020, when Covid-19 occurred, crude oil prices trended downwards, and from 2021 to 2022 crude oil prices had an upward trend with relatively high price fluctuations, and from 2023 to October the trend decreased and in November-December 2023 the trend increased. Figure 1(a) shows that movements in crude oil prices from 2017 to 2023 appear to indicate that prices are not stationary.

Figure 1(a) displays world crude oil prices, and Figure 1(b) shows world coal price data from 2017 to 2023. Data from 2017 to 2018 shows an upward trend and from 2019 to 2020

Table 6. Canonical Correlation Analysis

State Vector	Canonical Correlation	IC	Chi-Squares	DF
WOIL_P, WCOAL_P, WOIL_P$_{t-1\|t}$	1, 1, 0.1916	25.1562	64.7822	20
WOIL_P, WCOAL_P, WOIL_P$_{t-1\|t}$, WCOAL_P$_{t-1\|t}$	1, 1, 0.2099, 0.1478	0.4841	38.2743	19
WOIL_P, WCOAL_P, WOIL_P$_{t-1\|t}$, WOIL_P$_{t-2\|t}$	1, 1, 0.2222, 0.1528	3.2025	40.9978	18
WOIL_P, WCOAL_P, WOIL_P$_{t-1\|t}$, WOIL_P$_{t-2\|t}$, WOIL_P$_{t-3\|t}$	1, 1, 0.2275, 0.1835, 0.1341	-4.3925	31.4442	18

Table 7. The Parameters Estimate of the State-Space Model

Parameter	Estimate	Standard Error	t-value	Parameter	Estimate	Standard Error	t-value
$F(2,1)$	-3.0557	0.1733	-17.62	$F(4,4)$	0.0087	0.0844	0.10
$F(2,2)$	0.9921	0.0031	320.06	$G(3,1)$	1.0193	0.0137	73.90
$F(2,3)$	3.1154	0.1733	17.97	$G(3,2)$	0.0012	0.0040	0.30
$F(4,1)$	0.7262	0.0796	9.11	$G(4,1)$	1.0175	0.0165	61.43
$F(4,2)$	0.0089	0.0089	3.13	$G(4,2)$	0.0174	0.0044	3.93
$F(4,3)$	0.2552	0.0728	3.50				

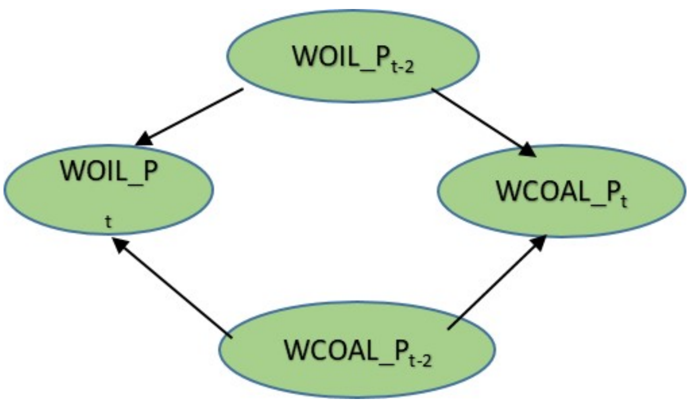

Figure 2. The Arrows ($Z \rightarrow T$) Indicate Significant Effect from Z to T

a downward trend appears. From 2021 to 2022, the world coal price trend appears to be increasing with relatively high fluctuations, and in 2023 the trend will be decreasing. Figure 1(b) World coal prices from 2017 to 2023 appear to indicate that they are not stationary.

3.1 Unit Root Test (ADF-test)

Table 1 displays the ADF test results before and after differencing, and it shows that world crude oil prices and world coal prices from 2017 to 2023 are not stationary and these outcomes are in accordance with the outcomes in Figure 1a and also in 1b Mireover, the test shows that the data is not stationary. The outcomes show that p-value for the ADF test before differencing is more than 0.05. This shows that the the is no evidence to reject null hypothesis. So, we are able to conclude that the data for both crude oil prices and world coal prices from 2017 to 2023 are not stationary. These results suggest that countries that are highly dependent on oil and coal exports will be vulnerable to prolonged price fluctuations that are influenced by many factors. Therefore, it is very important for countries that depend heavily on oil and coal exports to develop sustainable energy planning and management strategies to overcome continuous price fluctuations. Besides that, with data showing non-stationary and high price fluctuations, this has the implication that if a price shock occurs, the impact will take a long time to reach stability (Parreno, 2024). Therefore, many government leaders who have a role as decision makers must be able to plan adaptive energy policies to answer urgent needs and non-stationary energy consumption in the long term (Parreno, 2024).

From the ADF test results after differencing once (d=1) with the null hypothesis, the data is stationary, and H_0 is rejected with a p-value <0.0001. Table 2 displays the analysis results that up to lag-20 there appears to be a cross-correlation. The cross-correlation analysis results are presented in the Table 2.

In Table 2, the + sign indicates that at this lag there is a significant positive cross-correlation with a p-value < 0.05; the – sign indicates that at this lag there is a significant negative cross-correlation with a p-value < 0.05. By fulfilling the basic assumption that the data is stationary and there is cross correlation, then the time series vector of world crude oil prices (WOIL_P) and world coal prices (WCOAL_P) up to lag-20, the modeling can use a multivariate time series analysis approach. Table 3 displays the AIC analysis. It shows that the minimum AIC value is at AR2 (4.7515). Thus, based on these findings, the model that will be employed is VAR with order-2 [VAR(2)].

3.2 Model VAR(2) and its Estimates

Equation 17 shows the Model Multivariate Time Series for WOIL_P and WCOAL_P and Equation 18 shows the estimate

Table 8. Forecasting World Crude Oil Price (WOIL_P) and World Coal Price for the Next 30 Periods

No	Forecast WOIL_P	STD	Forecast WCOAL_P	STD
1743	90.6971	1.7944	161.251	6.5953
1744	90.7617	2.5632	162.878	9.3333
1745	90.6579	3.1591	163.972	11.6095
1746	90.6163	3.6341	165.244	13.3985
1747	90.6509	4.0681	166.742	15.0160
1748	90.5749	4.4614	167.885	16.5488
1749	90.5642	4.8116	169.218	17.8996
1750	90.5832	5.1473	170.633	19.1982
1751	90.5357	5.4616	171.830	20.4499
1752	90.5442	5.7539	173.190	21.6079
1753	90.5585	6.0386	174.558	22.7448
1754	90.5370	6.3093	175.804	23.8473
1755	90.5588	6.5674	177.174	24.8972
1756	90.5761	6.8203	178.521	25.9364
1757	90.5773	7.0635	179.807	26.9494
1758	90.6098	7.2989	181.182	27.9319
1759	90.6349	7.5303	182.525	28.9077
1760	90.6557	7.7549	183.845	29.8639
1761	90.6982	7.9744	185.223	30.8026
1762	90.7341	8.1908	186.573	31.7363
1763	90.7722	8.4022	187.921	32.6562
1764	90.8248	8.6103	189.306	33.5660
1765	90.8731	8.8160	190.670	34.4723
1766	90.9266	9.0181	192.043	35.3693
1767	90.9899	9.2180	193.438	36.2611
1768	91.0514	9.4159	194.821	37.1506
1769	91.1192	9.6114	196.216	38.0344
1770	91.1939	9.8055	197.625	38.9161
1771	91.2688	9.9981	199.029	39.7967
1772	91.3503	10.1891	200.446	40.6747

model.

$$\begin{bmatrix} \text{WOIL\_P}_t \\ \text{WCOAL\_P}_t \end{bmatrix} = \Phi_0 + \Phi_1 \begin{bmatrix} \text{WOIL\_P}_{t-1} \\ \text{WCOAL\_P}_{t-1} \end{bmatrix} + \Phi_2 \begin{bmatrix} \text{WOIL\_P}_{t-2} \\ \text{WCOAL\_P}_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} \text{WOIL\_P}_t \\ \text{WCOAL\_P}_t \end{bmatrix} = \begin{bmatrix} 0.0218 \\ 0.0359 \end{bmatrix} + \begin{bmatrix} 0.0156 & -0.0057 \\ 0.0025 & -0.0268 \end{bmatrix} \times \begin{bmatrix} \text{WOIL\_P}_{t-1} \\ \text{WCOAL\_P}_{t-1} \end{bmatrix} \quad (18)$$

Equation 19 displays the covariance innovation as follow:

$$\text{Cov} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} = \begin{bmatrix} 2.7802 & 1.2108 \\ 1.2108 & 41.9259 \end{bmatrix} \quad (19)$$

Table 4 displays the model parameter estimates and testing for WOIL_P and WCOAL_P.

Figure 2 and Table 4 show that the crude oil prices at time t (WOIL_P_t) is significantly affected by crude oil price at time $t-2$ (WOIL_P_{t-2}) with p-value = 0.0022, and by coal price at time $t-2$ (WCOAL_P_{t-2}) with p-value = 0.0001; the world coal price at time t (WCOAL_P_t) is significantly influenced by crude oil price at time $t-2$ (WOIL_P_{t-2}) with p-value = 0.0492, and by coal price at time $t-2$ (WCOAL_P_{t-2}) with p-value = 0.0001.

3.3 Granger Causality Test

Table 5 displays the Granger-causality test results of the relationship between world coal and crude oil prices. Test 1 with null hypothesis World crude oil price (WOIL_P) is affected by itself and not affected by past evidence of world coal price (WCOAL_P). Chi-Squares test results = 16.82 with p-value = $0.0008 < 0.05$, so reject the null hypothesis. We are able to conclude that the price of crude oil at time t not only depends on previous crude oil price information, but also depends on the price of coal in the past. This is in accordance with the results from Table 4 and Figure 2. Test 2 with null hypothesis World coal price (WCOAL_P) is affected by itself and not affected by past evidence of world crude oil price (WOIL_P). Chi-Squares test results = 14.89 with p-value = $0.0019 < 0.05$, so reject the null hypothesis. Thus, we are able to conclude that the coal price at time t not only depends on previous coal price information, but also depends on crude oil price in the past. This is in accordance with the results from Table 4 and Figure 2.

3.4 Canonical Correlation Analysis

Based on the results shown in Table 4, which examines the relationship between world crude oil prices (WOIL_P) and world coal prices (WCOAL_P), the optimal model identified was VAR(2). Using this VAR(2) model, a state-space forecasting model was subsequently developed.

To determine the state vector, the canonical correlation suggested by Akaike (1976) and Wei (2006) is used. The state vector is computed based on the Information Criterion (IC). If the IC value is negative, the minimal canonical correlation value is regarded zero (Wei, 2006); otherwise, it is > 0. For the first step, from Table 6, set of state vectors: WOIL_P_t, WCOAL_P_t, $\text{WOIL\_P}_{t+1|t}$. For this set of state vectors, the IC value is 25.1562, thus we enter the variable $\text{WOIL\_P}_{t+1|t}$ in state vectors. In the second step, set of state vectors: WOIL_P_t, WCOAL_P_t, $\text{WOIL\_P}_{t+1|t}$, $\text{WCOAL\_P}_{t+1|t}$. For this set of state vectors, the IC value is 0.4841, thus we enter the variable $\text{WCOAL\_P}_{t+1|t}$ in state vector. The third step, we have the set of state vectors: WOIL_P_t, WCOAL_P_t, $\text{WOIL\_P}_{t+1|t}$, $\text{WOIL\_P}_{t+2|t}$. For this set of state vectors, the IC value is 3.2025, thus we enter the variable $\text{WOIL\_P}_{t+2|t}$ in state vector. The fourth step, the set of state vectors: WOIL_P_t, WCOAL_P t, $\text{WOIL\_P}_{t+1|t}$, $\text{WOIL\_P}_{t+2|t}$, $\text{WOIL\_P}_{t+3|t}$. For this set of state vectors, the IC value is −4.3925, so we remove $\text{WOIL\_P}_{t+3|t}$ from the set of state vectors. Equation 20 displays the components of the state vector as follow:

$$Y_t = \begin{bmatrix} \text{WOIL\_P}_{t|t} \\ \text{WCOAL\_P}_{t|t} \\ \text{WOIL\_P}_{t+1|t} \\ \text{WOIL\_P}_{t+2|t} \end{bmatrix} \tag{20}$$

Table 6 shows the canonical correlation analysis, while Table 7 displays the parameters estimate of the state-space model.

Table 6 shows the outcomes of canonical correlation analysis, and Table 7 shows the parameters estimate of transition matrix F, and input matrix G. The model is:

$$Y_{t+1} = FY_t + GX_{t+1}$$

Equation 21 is the estimates state-space model and Equation 22 is the variance as shown as following:

$$\begin{bmatrix} \text{WOIL\_P}_{t+1|t+1} \\ \text{WCOAL\_P}_{t+1|t+1} \\ \text{WOIL\_P}_{t+2|t+1} \\ \text{WOIL\_P}_{t+3|t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -3.0557 & 0.9921 & 3.1154 & 0 \\ 0 & 0 & 0 & 1 \\ 0.7262 & 0.0089 & 0.2552 & 0.0087 \end{bmatrix} \begin{bmatrix} \text{WOIL\_P}_{t|t} \\ \text{WCOAL\_P}_{t|t} \\ \text{WOIL\_P}_{t+1|t} \\ \text{WOIL\_P}_{t+2|t} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1.0193 & 0.0042 \\ 1.0175 & 0.0074 \end{bmatrix} \begin{pmatrix} \varepsilon_{t+1} \\ \varphi_{t+1} \end{pmatrix} \tag{21}$$

$$\text{Var} \begin{pmatrix} \alpha_{t+1} \\ \beta_{t+1} \end{pmatrix} = \begin{bmatrix} 3.2199 & 1.7687 \\ 1.7687 & 43.4978 \end{bmatrix} \tag{22}$$

4. CONCLUSIONS

World crude oil and coal prices are interesting to analyze because they have a lot of influence on people's livelihoods and industry in many countries. In addition, significant price changes in these two commodities influence other commodity prices, the population's cost of living, and have a strong influence on a country's economic planning. World crude oil and coal price data is not stationary, so if a price shock occurs it will have a long-lasting effect. The best relationship pattern between world crude oil and coal prices is the VAR with order 2 VAR(2) model. From the results of the Granger-causality test analysis, crude oil and coal prices have a significant influence on each other, namely that crude oil prices are not only influenced by past evidence on crude oil prices, but are also influenced by past evidence on world coal prices. Likewise, coal prices are not only influenced by past evidence on coal prices, but are also influenced by past evidence on world crude oil prices. The results of forecasting analysis for world crude oil and coal prices for the next 30 periods using a state-space model approach, display that world crude oil prices tend to have a small increase; while forecasting coal prices for the next 30 periods tend to have an increasing trend.

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