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Research Paper

Utilization of Beta, Sigmoid, and Linear Fuzzy Membership Functions Discretization to Classify AISI 1045 Surface Roughness Levels Using the Ensemble of Multiple Naïve Bayes

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Abstract
Currently, researchers in various fields are using fuzzy discretization for decision-making. Discretization construction for numerical data involving a combination of fuzzy membership functions is significant because it can affect the performance of the model used. Classification of AISI 1045 surface roughness with satisfactory performance is needed to improve efficiency and extend the service life of AISI 1045 products. This study utilizes three fuzzy membership functions: beta, sigmoid, and linear in constructing fuzzy discretization on machining factors and tangential roughness levels to classify the axial roughness level of AISI 1045. Classification is performed using an ensemble of single naïve Bayes methods integrated with fuzzy discretization. These single methods are distinguished based on the combination of fuzzy membership functions used in the discretization. The results of the study show that the integration of fuzzy discretization through a combination of fuzzy membership functions, namely beta, sigmoid, linear, and fellow beta functions in the MNB method provides different performance, even the performance of MNB with fuzzy discretization using a combination of beta and sigmoid is almost the same or not statistically significantly different from the performance of the ensemble method. However, the ensemble method built provides the best performance for classifying the surface roughness level of AISI 1045, with Accuracy, Precision, Recall, F1-score, AUC, Balanced Accuracy, and G-Mean of 85.42%, 55.33%, 73.14%, 62.63%, 71.71%, 81.71%, and 81.04%, respectively.

Keywords
AISI 1045, Ensemble, Fuzzy, Multiple Naïve Bayes, Surface Roughness Levels

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1. INTRODUCTION

Zadeh introduced fuzzy sets in 1965 as a mathematical representation of ambiguities in everyday life, and they are a generalization of crisp set theory (Köseoğlu et al., 2024; Palanikumar et al., 2024). In a fuzzy set, the membership of an element is expressed in a degree that has a value between 0 and 1, and the function that represents the membership of the set in the interval [0,1] is called the fuzzy membership function (Hasan and Sobhan, 2020). This function generalizes the indicator function for the crisp set (Janková and Rakovská, 2022).

In carrying out prediction or classification tasks using fuzzy set theory, in addition to the mathematical or statistical model used, fuzzy membership functions also play a vital role (García-Zamora et al., 2024; Imani et al., 2021; Saatchi, 2024). Fuzzy membership function that represents a straight line on the coordinate plane, or what is called a linear fuzzy membership function (Campión et al., 2018; Michael, 2020; Sani, 2019), have several types, such as increasing-linear, decreasing-linear, triangular, and trapezoidal, and have been widely applied in many methods and cases (Resti et al., 2023; Yani et al., 2025a,b). On the other hand, if the representation in coordinate space is not a straight line, it is called a nonlinear fuzzy membership function (Bhattacharyya and Mukherjee, 2020; Wang, 2015). Some functions that fall into the nonlinear category are beta (Agung et al., 2009; Kresnawati et al., 2024), Gaussian (Muludi et al., 2024; Resti et al., 2025; Yani et al., 2025a), ascending sigmoid, and descending sigmoid (Bhattacharyya and Mukherjee, 2020; Kresnawati et al., 2024; Rutkowski, 2004; Medasani et al., 1998).

Research Yani et al. (2025a) that implemented a combination of fuzzy membership functions in the Decision Tree (DT) method to classify plastic waste types noted that the combination of linear fuzzy membership functions, namely decreasing-linear, triangular, and increasing-linear, provided the best performance compared to other combinations, including compared to the combination of Gaussian membership functions. In line with this research, research Resti et al. (2025) that classified surface roughness levels also found that the combination of fuzzy membership functions that provided the best performance was a linear combination. Specifically, the combination was decreasing-linear, trapezoidal, and increasing-linear. Similarly, the results of research Resti et al. (2023) classified corn pests and diseases and noted that the combination of linear fuzzy membership functions, namely decreasing-linear, triangular, and increasing-linear, was the combination that provided the best performance compared to other linear combinations, including combinations with nonlinear membership functions.

However, research Kresnawati et al. (2024) noted that a combination of membership functions with all nonlinear functions, namely beta, decreasing, and increasing sigmoid (diabetes mellitus-disease classification), provided better performance than a combination with all linear functions or a combination of linear and nonlinear functions. Furthermore, excessive linearity in some cases sometimes leads to errors (Bhattacharyya and Mukherjee, 2020).

In the manufacturing industry, AISI 1045 is widely used to make vehicle components for land, sea, and air applications that require different levels of roughness (Ijaz Malik et al., 2023; Siripath et al., 2024). The surface roughness of steel refers to the minute, microscopic wrinkles on its surface, resulting from the interaction between the cutting tool and the material (Tzotzis et al., 2025). Several factors, including cutting speed, feed rate, and depth of cut (Guo et al., 2024; Pimenov et al., 2025; Praveen et al., 2025; Qasim et al., 2015), influence the surface roughness level in the manufacturing process (Abellán-Nebot et al., 2024; Thamrin, 2019; Trung, 2020; Yanis et al., 2019).

Surface roughness is critical in many applications and can be used as a benchmark for assessing machined products (La Fé-Perdomo et al., 2023; Otsuki et al., 2022; Ruzova and Haddadi, 2025; Sukindar et al., 2024). Generally, steel surface roughness is measured experimentally. However, prediction or classification based on experimental results can be an effective alternative if it achieves satisfactory performance, given that experiments are limited due to high costs and inefficiencies (Balasuadhakar et al., 2025; Dubey et al., 2022; Ross et al., 2024). The ensemble method combines several single methods to achieve better performance through a voting majority system (An et al., 2024; Ganaie et al., 2022; Lu et al., 2023; Mohammed and Kora, 2023; Zhao and Ye, 2024). However, not all ensemble methods that are built successfully have better performance than the single methods that build them, including those built from fuzzy discretization (Yani et al., 2025a; Resti et al., 2025).

Classifying AISI 1045 surface roughness with satisfactory performance is needed to improve efficiency and extend the service life of AISI 1045 products (Alajmi and Almeshal, 2021; Baldin et al., 2023; Chen et al., 2024; Stojković et al., 2022). In this study, we propose an ensemble method based on multiple naïve Bayes classifiers to classify AISI 1045 surface roughness. Each single naïve Bayes method is integrated with fuzzy discretizations of different membership functions: beta (Kresnawati et al., 2024), sigmoid (Bhattacharyya and Mukherjee, 2020; Kresnawati et al., 2024; Rutkowski, 2004; Medasani et al., 1998), and linear, especially decreasing and increasing linear (Fernandez et al., 2022; Resti et al., 2025; Yani et al., 2025a,b). These functions were chosen because each has different representation characteristics: the beta function captures flexible variations in distribution shapes, the sigmoid function is effective for modeling gradual transitions, and the linear function provides a simple representation at interval boundaries. The integration of these three functions enables a more adaptive representation of data ambiguity than conventional fuzzy discretization approaches. In addition, this study also wants to show that the multiple Naïve Bayes ensemble method has better performance than the naïve Bayes methods that build it, because not all ensemble methods built from Single methods provide better performance than the Single methods themselves, even though the initial goal of building the ensemble method was to obtain better performance.

2. EXPERIMENTAL SECTION

This research consisted of two main stages: an experiment measuring the axial surface roughness of milled AISI 1045 and a classification using an ensemble method developed from multiple naïve Bayesian algorithms with fuzzy membership functions: beta, sigmoid, and linear. The experiment measured the axial surface roughness of an AISI 1045 specimen measuring 200 mm × 100 mm × 25 mm, with a hardness of 40 – 45 HRC. Milling was performed using an OPTImill F 105 CNC milling machine with a wet machining system using Bromus. The results of axial roughness measurements are converted according to the levels specified in ISO 4948. The machining factors: cutting speed (V_c), feed rate (f_z), depth of cut (a_x) were set to 6, 5, and 4 levels, respectively. The cutting tool on the milling machine had a coated carbide tip with a hardness of 90-93 HRC. The cutting method was performed on the surface in the downturn direction. The surface roughness of the product was measured using a Handysurf Accretech E-35B instrument with a measuring pattern, an accuracy of 0.01 μm, a measuring length of 4 mm, and a cutting limit of 0.8. The tangential surface roughness is measured at six points (R_{t_1}–R_{t_6}) by considering the representative and random principles (Resti et al., 2025).

The classification stages using the ensemble method built from multiple naïve Bayes are presented in Figure 1. Three combinations of fuzzy membership functions are used to build fuzzy discretization, and each is integrated with the multinomial naïve Bayes method to produce three different multinomial naïve Bayes methods. The three combinations consist of (1) a decreasing-increasing sigmoid function with a beta function, (2) a decreasing-increasing linear function with a beta function, and (3) a beta function with each beta. The MNB methods integrated with the discretization of each of these combinations are named MNB1, MNB2, and MNB3, respectively. The sampling process in building the ensemble method uses bootstrap resampling. This method automatically generates diverse models, eliminating the need for specific model design (Duran-Rosal et al., 2025). The final results were obtained using the majority voting system without weighting to maintain model simplicity, as simple ensemble models tend to be more robust and easier to implement (Wang et al., 2024).

Fuzzy discretization is a way of mapping the continuous domain of a variable into several fuzzy sets represented by certain membership functions, so that one value can be associated with more than one linguistic interval through the degree of membership in [0, 1], or it can also be interpreted as a technique for transforming continuous data into some linguistic intervals using the concept of fuzzy sets, where each value is not placed strictly in one interval, but can have a degree of membership in one or more intervals at once (Resti et al., 2023). In more formal language, let X_d be the universal set, $\tilde{X}_d$ be the fuzzy set is obtained from X_d, namely a linguistic set written as,

$$\tilde{X}_d = \{\tilde{X}_{d_1}, \tilde{X}_{d_2}, \cdots, \tilde{X}_{d_c}\} \tag{1}$$

Each value of x_f is fuzzy discretized into:

$$FD = \{\mu_{\tilde{X}_{d_1}}(x_f), \mu_{\tilde{X}_{d_2}}(x_f), \cdots, \mu_{\tilde{X}_{d_c}}(x_f)\} \tag{2}$$

where $\mu_{\tilde{X}_{d_k}}$ is the membership function of the k-th linguistic, $k = 1, 2, \cdots, c$, $x_f \in X$, $X \subseteq \mathbb{R}$, and x_f is continuous.

Each membership function in one variable can be different and combined (Algehyne et al., 2022; Chen and Huang, 2021; Dai et al., 2024; Fernandez et al., 2022; Yani et al., 2025a). Three ensemble methods are proposed, built from MNB methods integrated with fuzzy discretization using three different combinations of membership functions: beta, sigmoid, and linear. For the sigmoid and the linear functions, the decreasing and increasing types are used because they are not equivalent functions.

The formula for each function beta, decreasing and increasing of sigmoid, and decreasing and increasing of linear is given in García-Zamora et al. (2024); Hasan and Sobhan (2020); Imani et al. (2021); Janková and Rakovská (2022); Kresnawati et al. (2024); Resti et al. (2023); Saatchi (2024). The beta fuzzy membership function, as presented in Equation (Hasan & Sobhan, 2020), has a parameter γ, a domain element with a membership value of 1, and is also the center of the curve. For the decreasing sigmoid as written in Janková and Rakovská (2022), let α be the smallest domain element, which has a membership degree of 1, β be the domain element at an inflection point, which has a membership degree of 0.5, and γ be the most prominent domain element, which has a membership degree of 0. In contrast, the definition for the increasing sigmoid function in García-Zamora et al. (2024), α has a membership degree of 0, and γ has a membership degree of 1, where β also has a membership degree of 0.5. For the decreasing linear in Saatchi (2024), a is the smallest domain element with a membership degree of 1, and b is the most prominent domain element with a membership degree of 0. In contrast, for the increasing linear in Imani et al. (2021), a has a membership degree of 0, and b has a membership degree of 1.

$$\mu_{\tilde{X}_d}(x_f; \gamma, \beta) = \frac{1}{1 + \left(\frac{x_f - \gamma}{\beta}\right)^2}, \qquad \gamma - \beta \le x_f \le \gamma + \beta \tag{3}$$

$$\mu_{\tilde{X}_d}(x_f; \alpha, \beta, \gamma) = \begin{cases} 1, & x_f \le \alpha \\ 1 - 2\left(\frac{x_f - \alpha}{\beta - \alpha}\right)^2, & \alpha \le x_f \le \beta \\ 2\left(\frac{\gamma - x_f}{\gamma - \beta}\right)^2, & \beta \le x_f \le \gamma \\ 0, & x_f \ge \gamma \end{cases} \tag{4}$$

$$\mu_{\tilde{X}_d}(x_f; \alpha, \beta, \gamma) = \begin{cases} 0, & x_f \le \alpha \\ 2\left(\frac{x_f - \alpha}{\beta - \alpha}\right)^2, & \alpha \le x_f \le \beta \\ 1 - 2\left(\frac{\gamma - x_f}{\gamma - \beta}\right)^2, & \beta \le x_f \le \gamma \\ 1, & x_f \ge \gamma \end{cases} \tag{5}$$

$$\mu_{\tilde{X}_d}(x_f; a, b) = \begin{cases} 1, & x_f \le a \\ \dfrac{b - x_f}{b - a}, & a \le x_f \le b \\ 0, & x_f \ge b \end{cases} \tag{6}$$

$$\mu_{\tilde{X}_d}(x_f; a, b) = \begin{cases} 0, & x_f \le a \\ \dfrac{x_f - a}{b - a}, & a \le x_f \le b \\ 1, & x_f \ge b \end{cases} \tag{7}$$

Three different combinations of fuzzy membership functions are constructed from crisp discretization, yielding five linguistic representations for each predictor variable. The crisp discretization for each predictor is formed using (8), for X_d^o and X_d, are the d-th initial predictor variable and d-th discretized predictor variables, respectively (Resti et al., 2023).

$$X_d = X_d^o + \text{Range}(X_d^o) \tag{8}$$

Each of these three fuzzy membership combinations then differentiates the MNB methods. Statistical tests are presented to demonstrate that the methods differ and that the ensemble method's performance improves over the individual techniques that construct it. The parameters of each fuzzy membership function are obtained through the tuning system (Resti et al., 2023, 2025; Yani et al., 2025a,b).

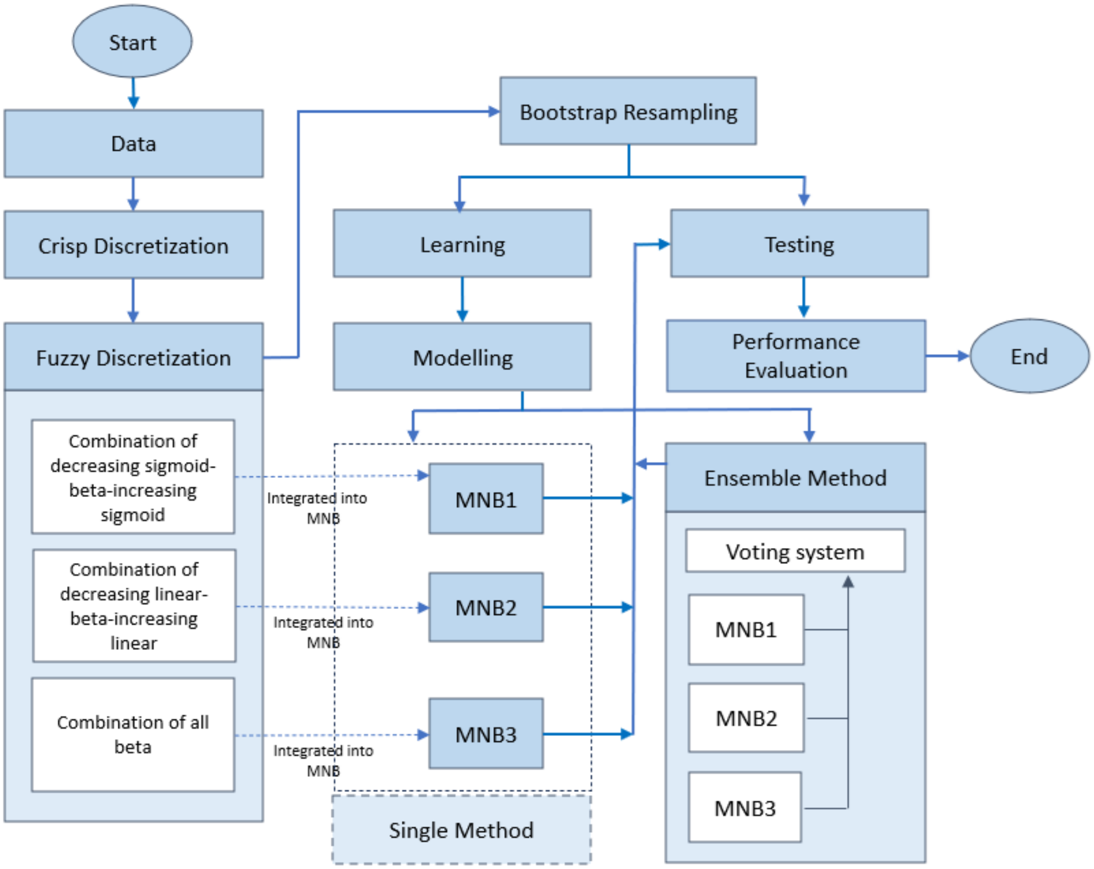

Figure 1. Flowchart of Proposed Method

Table 1. Statistics Summary of Each Class

Class (% sample distribution)	Statistics	V_c	f_z	a_x	R_{t_1}	R_{t_2}	R_{t_3}	R_{t_4}	R_{t_5}	R_{t_6}
1	Mean	13.75	0.09	1.38	0.60	0.60	0.55	0.52	0.59	1.20
(2%)	Stdev	8.84	0.02	0.18	0.00	0.14	0.21	0.25	0.16	0.57
2	Mean	14.26	0.07	1.19	3.34	3.41	3.42	2.78	3.27	3.19
(14%)	Stdev	5.06	0.03	0.30	1.77	2.17	1.47	1.63	2.02	1.97
3	Mean	13.75	0.09	1.11	5.39	5.41	4.77	4.66	4.76	5.99
(37%)	Stdev	4.59	0.04	0.27	1.68	3.25	1.54	1.70	1.65	6.01
4	Mean	13.65	0.10	1.11	7.68	7.48	7.46	7.13	7.32	7.68
(43%)	Stdev	3.91	0.04	0.29	2.73	2.39	2.31	1.78	1.70	2.06
5	Mean	13.00	0.12	1.10	10.14	10.54	9.08	9.57	10.74	9.86
(4%)	Stdev	1.12	0.04	0.29	2.71	2.69	1.86	3.56	4.99	2.08

3. RESULTS AND DISCUSSION

The class distribution and statistical summary of the machining factors and the tangential roughness level for each class are presented in Table 1.

Table 1 shows that the classification of axial surface roughness levels from AISI 1045 is a multiclass case with an unbalanced class distribution. The majority are in the third and fourth classes, and the first and fifth classes are the minority classes. Since the ensemble method is one of the methods capable of mitigating unbalanced data (Wainer, 2024), this study does not explicitly balance the classes. This study also measures the performance of the proposed model by involving metrics that cover class imbalance, namely Macro Average for Precision, Recall, F1-score, which give equal weight to each class, so that the minority class is not obscured by the majority class (De Angeli et al., 2022), also Balance Accuracy (BA), and G-Mean as evaluation metrics that balance between classes (Khan et al., 2024).

Regarding the values of the machining factors and the tangential roughness level, almost all variables are more dominant in class 5, which is indicated by the minimum standard deviation, except for the variables of cutting speed and depth of cut, which are more dominant in class 2 and class 1, respectively. The only consistent tangential surface roughness value is in class 1. The different means and standard deviations within each class also indicate that each class of axial surface roughness has distinct characteristics, suggesting that further analysis using the classification method is relevant.

To classify the level of axial surface roughness based on the predictor variables presented in Table 1, a discretization into five linguistic terms for each predictor variable is proposed. The division of each variable's value range into five categories is based on prior knowledge (Resti et al., 2023, 2025). Specifically, the linguistic terms corresponding to categories 1-5 for each variable are presented in Table 2.

In fuzzy discretization, each linguistic term in each variable corresponds to a level from the lowest to the highest value and is assumed to follow a membership function. The MNB1 model is a single classification model based on fuzzy discretization designed using a combination of membership functions that are all nonlinear, namely decreasing sigmoid – beta - increasing sigmoid, where the first two categories are represented by decreasing sigmoid functions, the third category by beta functions, and the last two categories by increasing sigmoid functions. Table 3 presents the parameters of the fuzzy membership function in the MNB1 model obtained through the tuning system.

In the MNB1 model, the parameter α in the first linguistic term and the parameter γ in the fifth linguistic term for each predictor variable are the lowest and highest values for that variable, respectively. The decreasing sigmoid function is used for linguistic terms at the beginning of the domain, as it provides a high degree of membership at small values and gradually decreases as the variable value increases. Conversely, the increasing sigmoid function is applied to linguistic terms at the end of the domain, as it can represent a gradual increase in the degree of membership at large values. Meanwhile, linguistic terms in the middle of the domain are modeled using the beta distribution, which is bell-shaped. This function is practical for representing values with a center (the most representative value) and straightforward lower and upper boundaries, making it suitable for describing symmetrically overlapping linguistic categories. The intervals listed in the table indicate the range of values within which a linguistic term has a dominant degree of membership. The overlap between intervals reflects the data's uncertainty and ambiguity, key characteristics of the fuzzy approach.

The MNB2 model is a single-classification model based on fuzzy discretization, using a combination of linear and nonlinear membership functions: linear descending- beta- linear increasing, where linear descending functions represent the first two categories, the third category is represented by beta functions, and the last two categories are defined by linear as-

Table 2. Linguistic Terms of Predictor Variables

Predictor Variable	Range	Category	Linguistic term
V_c	7.5–10 m/s	1	Very Slow
	10.1–12.6 m/s	2	Slow
	12.7–15.2 m/s	3	Medium
	15.3–17.8 m/s	4	Fast
	17.9–20.4 m/s	5	Very Fast
f_z	0–0.03 mm/gear	1	Very Slow
	0.04–0.07 mm/gear	2	Slow
	0.08–0.11 mm/gear	3	Medium
	0.12–0.15 mm/gear	4	Fast
	0.16–0.19 mm/gear	5	Very Fast
a_x	0.75–0.9 mm	1	Very Shallow
	0.91–1.06 mm	2	Shallow
	1.07–1.22 mm	3	Medium
	1.23–1.38 mm	4	Deep
	1.39–1.54 mm	5	Very Deep
R_{t_1}	0.4–3.36 μm	1	Very Rough
	3.37–6.33 μm	2	Rough
	6.34–9.3 μm	3	Medium
	9.4–12.36 μm	4	Smooth
	12.37–15.3 μm	5	Very Smooth
R_{t_2}	0.5–5.16 μm	1	Very Rough
	5.17–9.83 μm	2	Rough
	9.84–14.5 μm	3	Medium
	14.6–19.26 μm	4	Smooth
	19.27–23.9 μm	5	Very Smooth
R_{t_3}	0.4–3.6 μm	1	Very Rough
	3.7–6.9 μm	2	Rough
	7–10.2 μm	3	Medium
	10.3–13.5 μm	4	Smooth
	13.6–16.8 μm	5	Very Smooth
R_{t_4}	0.33–3.21 μm	1	Very Rough
	3.21–6.09 μm	2	Rough
	6.09–8.97 μm	3	Medium
	8.97–11.85 μm	4	Smooth
	11.85–14.72 μm	5	Very Smooth
R_{t_5}	7.5–10 μm	1	Very Rough
	10.1–12.6 μm	2	Rough
	12.7–15.2 μm	3	Medium
	15.3–17.8 μm	4	Smooth
	17.9–20.4 μm	5	Very Smooth
R_{t_6}	0–0.03 μm	1	Very Rough
	0.04–0.07 μm	2	Rough
	0.08–0.11 μm	3	Medium
	0.12–0.15 μm	4	Smooth
	0.16–0.19 μm	5	Very Smooth

Figure 2. The Proposed Method Performance

Table 3. Parameters of the Fuzzy Membership Function in the MNB1

Predictor variable	1 $[\alpha, \beta, \gamma]$	2 $[\gamma-\beta, \gamma, \gamma+\beta]$	3 $[\gamma-\beta, \gamma, \gamma+\beta]$	4 $[\gamma-\beta, \gamma, \gamma+\beta]$	5 $[\alpha, \beta, \gamma]$
V_c	[7.50,9.01,10.50]	[10.00,11.50,13.00]	[12.50,14.00,15.05]	[15.00,16.50,18.00]	[17.00,18.50,20.00]
f_z	[0.00,0.02,0.04]	[0.03,0.04,0.06]	[0.05,0.07,0.09]	[0.08,0.10,0.12]	[0.10,0.13,0.15]
a_x	[0.75,0.88,1.00]	[0.90,1.03,1.15]	[1.05,1.18,1.30]	[1.20,1.33,1.45]	[1.25,1.38,1.50]
Rt_1	[0.40,2.04,3.70]	[3.65,5.30,6.95]	[6.90,8.55,10.20]	[10.15,11.80,13.45]	[11.38,15.03,16.70]
Rt_2	[0.50,2.15,3.70]	[1.90,4.40,6.90]	[5.10,6.70,8.30]	[6.50,8.10,9.70]	[7.92,15.67,23.80]
Rt_3	[0.40,2.11,3.80]	[1.50,3.02,4.90]	[2.60,4.30,6.00]	[3.70,5.40,7.10]	[7.01,10.44,16.40]
Rt_4	[0.33,2.43,3.48]	[1.68,3.26,4.83]	[3.03,4.61,6.18]	[4.38,5.96,7.53]	[5.74,10.21,14.72]
Rt_5	[0.40,2.71,5.00]	[1.80,4.10,6.40]	[3.20,5.50,7.80]	[4.60,6.90,9.20]	[5.99,12.34,18.70]
Rt_6	[0.35,4.46,8.55]	[1.95,6.05,10.15]	[3.55,7.65,11.75]	[5.15,9.25,13.35]	[6.73,19.52,32.30]

Table 4. Parameters of the Fuzzy Membership Function in the MNB2

Predictor variable	1 $[\alpha, \beta]$	2 $[\gamma-\beta, \gamma, \gamma+\beta]$	3 $[\gamma-\beta, \gamma, \gamma+\beta]$	4 $[\gamma-\beta, \gamma, \gamma+\beta]$	5 $[\alpha, \beta]$
V_c	[7.50,10.00]	[8.80,10.70,12.60]	[11.60,13.40,15.20]	[14.30,16.05,17.80]	[16.93,20.00]
f_z	[0.00,0.03]	[0.02,0.04,0.06]	[0.05,0.07,0.09]	[0.08,0.09,0.12]	[0.11,0.15]
a_x	[0.75,0.90]	[0.83,0.94,1.06]	[1.00,1.11,1.22]	[1.17,1.27,1.38]	[1.33,1.50]
Rt_1	[0.40,3.66]	[2.03,4.46,6.93]	[5.71,7.95,10.20]	[9.08,11.32,13.56]	[12.44,16.70]
Rt_2	[0.50,5.16]	[2.83,6.33,9.83]	[8.08,11.29,14.50]	[12.90,16.08,19.26]	[17.67,23.80]
Rt_3	[0.40,3.60]	[2.00,4.45,6.90]	[5.68,7.94,10.20]	[9.07,11.30,13.50]	[12.40,16.40]
Rt_4	[0.33,3.21]	[1.77,3.93,6.09]	[5.01,6.99,8.97]	[7.98,9.91,11.85]	[10.88,14.72]
Rt_5	[0.40,4.06]	[2.23,4.98,7.73]	[6.36,8.88,11.40]	[10.14,12.65,15.16]	[13.90,18.70]
Rt_6	[0.35,6.74]	[3.55,8.34,13.14]	[10.74,15.14,19.54]	[17.34,21.64,25.94]	[23.79,32.30]

Table 5. Parameters of the Fuzzy Membership Function in the MNB3

Predictor variable	1 $[\gamma-\beta, \gamma, \gamma+\beta]$	2 $[\gamma-\beta, \gamma, \gamma+\beta]$	3 $[\gamma-\beta, \gamma, \gamma+\beta]$	4 $[\gamma-\beta, \gamma, \gamma+\beta]$	5 $[\gamma-\beta, \gamma, \gamma+\beta]$
V_c	[7.50,9.50,11.50]	[9.00,11.00,13.00]	[12.00,14.00,16.00]	[15.00,16.25,17.50]	[17.00,18.50,20.00]
f_z	[0.00,0.02,0.04]	[0.02,0.04,0.06]	[0.04,0.07,0.09]	[0.08,0.10,0.13]	[0.11,0.14,0.15]
a_x	[0.75,0.85,0.96]	[0.95,1.05,1.15]	[1.14,1.25,1.35]	[1.33,1.44,1.55]	[1.45,1.48,1.50]
Rt_1	[0.40,2.05,3.70]	[3.66,5.35,7.00]	[6.95,8.65,10.30]	[10.25,11.95,13.60]	[13.55,15.25,16.67]
Rt_2	[0.50,3.00,5.50]	[5.45,8.00,10.50]	[10.46,13.00,15.50]	[15.46,18.00,20.50]	[20.46,22.15,23.80]
Rt_3	[0.40,2.10,3.80]	[3.77,5.50,7.20]	[7.16,8.90,10.60]	[10.55,12.30,14.00]	[13.96,15.20,16.40]
Rt_4	[0.33,1.83,3.33]	[3.29,4.82,6.33]	[6.28,7.80,9.32]	[9.30,10.82,12.31]	[12.27,13.52,14.72]
Rt_5	[0.40,2.25,4.10]	[4.05,5.95,7.80]	[7.75,9.63,11.52]	[11.47,13.32,15.23]	[15.18,16.95,18.70]
Rt_6	[0.35,3.54,6.75]	[6.72,9.93,13.15]	[13.11,16.35,19.58]	[19.55,22.74,25.98]	[25.95,29.13,32.30]

cending functions. Table 4 displays the parameters of the fuzzy membership function in the MNB2 model, which were also produced by the tuning system.

Like the MNB1 model, in the MNB2 model, the first parameter in the first linguistic term (a) and the last parameter in the fifth linguistic term (b) for each predictor variable are the lowest and highest values for that variable, respectively. The use of decreasing linear, beta, and increasing linear membership functions in the formation of the MNB2 model was chosen as an alternative to the sigmoid function to yield a simpler, more interpretable fuzzy model. Compared to the sigmoid function, which has a nonlinear transition and tends to be smoother, the linear function represents changes in the degree of membership proportionally and directly to changes in variable values. The linear function has a simpler structure and is therefore more efficient in computation. In addition, the interpretation of the relationship between numerical values and linguistic terms becomes more intuitive because the change in the degree of membership occurs constantly.

In contrast, the sigmoid function is generally more appropriate when the data exhibit very gradual or nonlinear transitions, especially at the domain boundaries. However, in the

Table 6. Mean and Standard Deviation of Proposed Method Performance

Performance Metric	Statistic Parameter	MNB1	MNB2	MNB3	Ensemble
Accuracy	Mean	83.75	77.44	81.03	85.42
	Std	2.52	3.71	3.68	2.90
Precision	Mean	53.06	39.22	45.70	55.33
	Std	10.02	10.29	11.69	10.38
Recall	Mean	71.52	55.42	64.17	73.14
	Std	11.14	10.91	13.87	11.45
F1 score	Mean	60.51	45.59	53.02	62.63
	Std	9.36	10.18	11.86	9.84
AUC	Mean	69.92	59.73	64.62	71.71
	Std	5.58	6.30	7.03	5.77
Balanced Accuracy	Mean	80.34	70.19	75.76	81.71
	Std	6.09	6.12	7.80	6.28
G-Mean	Mean	79.64	68.34	74.50	81.04
	Std	6.74	7.17	8.93	6.92

Table 7. ANOVA of Four Proposed Multiple Naïve Bayes Methods

Metrics	Source of Var.	Sum of Squares	Mean Squares	F	p-Value	F-Criteria
Accuracy	between	1275.85	425.28	41.40	9.9×10^{-19}	
	within	1431.66	10.53			
Recall	between	6884.68	2294.89	16.20	4.6×10^{-9}	
	within	19262.80	141.64			
Precision	between	5526.58	1842.19	16.37	3.9×10^{-9}	2.67
	within	53061.10	112.55			
Specificity	between	567.05	189.02	40.40	9.9×10^{-19}	
	within	636.29	4.68			
F1 score	between	6240.03	2080.01	19.33	1.7×10^{-10}	
	within	14634.30	107.61			
AUC	between	3042.82	1014.27	26.42	1.6×10^{-13}	
	within	5221.90	38.40			
Balanced Accuracy	between	2484.26	948.09	21.68	1.6×10^{-11}	
	within	3948.62	43.74			
G-Mean	between	3585.04	1161.68	20.70	4.2×10^{-11}	
	within	7632.22	56.12			

context of this study, the assumption of linear change is considered adequate to represent the relationship between variable values, so selecting the linear function is sufficiently representative without significantly sacrificing model accuracy.

Meanwhile, the beta function is still used in the middle part of the linguistic terms because of its ability to model dominant central values and provide clear lower and upper boundaries. The combination of a linear function in the extreme domain and a beta function in the middle domain achieves a balance among model simplicity, representation flexibility, and the ability to handle data uncertainty. Thus, the selection of the linear–beta–linear membership function can be justified methodologically as an efficient, interpretable, and appropriate approach to support the fuzzy discretization process and the development of linguistic-based classification systems.

Table 5 shows the parameters of the fuzzy membership function in the MNB3 model, also obtained through the tuning system. The MNB3 model, like the MNB1 model, is a single-classification model based on fuzzy discretization, using a combination of all nonlinear membership functions, but beta functions represent all categories. Each beta function corresponds to the most dominant central value of each linguistic term and has clear lower and upper boundaries, allowing gradual transitions between categories. The value intervals in the table indicate the dominance range of each linguistic term, with overlaps reflecting the data's uncertainty and ambiguity. This approach produces a balanced and consistent linguistic representation across variable domains and is suitable for supporting

Table 8. Tukey-Kramer Post Hoc Test

Proposed Models Comparison	Accuracy	Precision	Recall	F1 score	AUC	BA	GM
MNB 1 vs MNB 2	6.31	13.48	16.10	14.72	10.00	10.15	11.31
MNB 1 vs MNB 3	2.72	7.00	7.35	7.29	5.12	4.58	5.15
MNB 1 vs ensemble	1.67	2.63	1.62	2.32	1.97	1.37	1.40
MNB 2 vs MNB 3	3.59	6.48	8.75	7.43	4.89	5.57	6.16
MNB 2 vs ensemble	7.98	16.11	17.72	17.04	11.08	11.52	12.70
MNB 3 vs ensemble	4.39	9.63	8.97	9.61	7.09	5.95	6.54

Table 9. Performance Comparison of Models without and with Fuzzy Discretization

Author	Model	Accu	Prec	Recall	F1s	AUC	BA	GM
(Resti et al., 2025)	DT1 (without fuzzy discretization)	78.50	47.36	67.25	55.39	66.28	-	-
	DT2	74.02	46.02	54.42	49.87	64.05	-	-
	DT3	75.34	40.05	53.48	45.79	61.10	-	-
	DT4	80.73	48.53	73.47	58.44	67.72	-	-
	Ensemble of DT1, DT2, DT3	73.33	35.22	36.79	35.99	58.02	-	-
	Ensemble of DT1, DT2, DT4	82.64	51.62	53.72	52.65	69.62	-	-
	Ensemble of DT1, DT3, DT4	81.33	51.62	56.29	53.86	69.04	-	-
	Ensemble of DT2, DT3, DT4	82.67	55.19	53.97	54.57	71.02	-	-
Proposed Method	MNB (without fuzzy discretization)	71.37	37.50	39.01	37.46	57.31	59.96	55.36
	MNB1	83.75	52.70	71.52	60.31	69.74	80.34	79.64
	MNB2	77.44	39.22	55.42	45.59	59.73	70.19	68.34
	MNB3	81.03	45.70	64.17	53.02	64.62	75.76	74.50
	Ensemble of MNB1, MNB2, MNB3	85.42	55.33	73.14	62.63	71.71	81.71	81.04

fuzzy discretization for classification models. In this model, because all of them use the beta function, the lowest and highest values of each linguistic term are the subtraction and addition of the values of the parameters γ and β.

The performance of each of the proposed models (3 single MNB models and the ensemble) for each of the 35 bootstrap resamplings is presented in Figure 2. The performance of all four proposed models does not include a confusion matrix, because each bootstrap in a given model produces a single confusion matrix; across 35 bootstrap resamplings, this study has 140 confusion matrices.

In general, across all bootstrap sampling methods, except Accuracy (which is not a suitable metric for imbalanced classes), the Balanced Accuracy metric yields the highest value for all four proposed methods, followed by G-Mean, Recall, AUC, F1-score, and Precision. The mean and standard deviation of all bootstrap sampling for each metric of the four proposed methods are presented in Table 6. The ensemble method has the highest average performance compared to the three Single MNB methods that compose it. Among the three single methods, MNB1 has the highest average performance, while MNB2 has the lowest. The AUC metric (except Accuracy) has the lowest sensitivity to data changes among all proposed methods, followed by Balanced Accuracy, G-Mean, F1 score, Precision, and Recall.

The average values of the metrics in MNB1 and the ensemble tend to be similar, but they tend to differ between MNBs, as do their standard deviations. The results of hypothesis testing for the performance of the four proposed methods, which vary from one another, are presented in Tables 7 and 8.

With an F-criterion of 2.67 and a p-value below α, the four methods differ significantly in overall performance. However, the pair of methods whose performance differs significantly is evident in the post hoc test results, as shown in Table 8.

With the critical Q values for the five metrics being 1.49, 5.48, 4.88, 4.77, and 2.85, it can be concluded that only the MNB 1 and ensemble pairs are not significantly different. In contrast, the other five pairs are significantly different. The integration of fuzzy discretization through a combination of fuzzy membership functions, namely beta, sigmoid, linear, and fellow beta functions in the MNB method, provides different performance; even the performance of MNB1 (fuzzy discretization using a combination of beta and sigmoid) is almost the same or not statistically significantly different from the performance of the ensemble method. However, the performance of the constructed ensemble method is better.

Not all ensemble methods built from single methods perform better than the single methods themselves; for example, research (Resti et al., 2025). In the research, which also predicted the surface roughness of AISI 1045 using an ensemble method built from multiple decision tree methods based on fuzzy discretization, noted that the ensemble performance was better than the performance of the single methods that composed it. The fuzzy discretization in the multiple decision tree methods was designed from a combination of linear and Gaussian fuzzy membership functions. The DT method without fuzzy discretization (DT1) was used to construct the ensemble method because not all DT methods with fuzzy discretization performed better than DT1. DT2 and DT3 are DT methods that combine linear-Gaussian and linear-triangular models, respectively. Only DT4 (a combination of linear and trapezoidal), a model with fuzzy discretization, performed better than DT1. Similarly, not all ensemble methods constructed from multiple DTs performed better than the single DT methods that constructed them. However, the multiple DT ensemble method constructed from DT methods with fuzzy discretization (Ensemble of DT2, DT3, DT4) performed the best. A comparison of the performance of the methods, with and without fuzzy discretization, including the ensemble method, is presented in Table 9.

From Table 9, it can also be seen that, in this proposed research, the performance of all MNB methods with fuzzy discretization is better than that of the MNB method without fuzzy discretization. This fact is why the proposed ensemble method is built solely from MNB methods with fuzzy discretization, without including any single method without fuzzy discretization. This research also shows that the ensemble method of multiple Naïve Bayes, built from naïve Bayes methods with fuzzy discretization, especially from three combinations of MNB1, MNB2, and MNB3, performs better than the single methods that build it.

4. CONCLUSIONS

This study has developed an ensemble method of multiple naïve Bayes to classify the axial AISI 1045 surface roughness. Each naïve Bayes method is integrated with fuzzy discretizations using the beta, sigmoid, and linear membership functions. Specifically, it is integrated with combinations of decreasing sigmoid, beta, and increasing sigmoid (MNB1), decreasing linear, beta, and increasing linear (MNB2), and all beta functions (MNB3). Based on statistical tests, the four proposed methods are generally significantly different, but the pair of MNB 1 and the ensemble methods is not significantly different. MNB 1 performs best among MNB 2 and MNB 3. However, in general, the ensemble method built performs best across all metrics measured. This study also shows that the ensemble method of multiple Naïve Bayes performs better than the naïve Bayes methods that build it, resulting from integration with fuzzy discretization. The ensemble performance is the best for classifying the surface roughness level of AISI 1045, with Accuracy, precision, recall, F1-score, AUC, Balanced Accuracy, and G-Mean of 85.42%, 55.33%, 73.14%, 62.63%, 71.71%, 81.71%, and 81.04%, respectively. The research results, in the form of an axial surface roughness classification derived from this experiment, are expected to serve as an alternative, given that high costs and inefficiency limit experimentation.

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