Science and Technology Indonesia
e-ISSN:2580-4391 *p*-ISSN:2580-4405
Vol. 7, No. 3, July 2022

Research Paper

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Evaluate All The Order of Every Element in The Higher Even, Odd, and Prime Order of Group for Composition

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Abstract
This paper aims to treat a study on the order of every element in the higher even, odd and prime order of group for composition. In fact, express order of a group and order of an element of a group in real numbers. Here we discuss the higher order of groups in different types of order, which will give us practical knowledge to see the applications of the composition. In order to find out the order of an element $a^m \in G$ in which $a^n = e$= identity element, then find the least common multiple (*i.e.*(LCM))= λ) of m and n. The least common multiple of two numbers is the "smallest non-zero common number," which is a multiple of both the numbers. So $O(a^m)$= λ/m. Also, if G is a finite group, n is a positive integer, and $a \in G$ then the order of the products *na*. When G is a finite group, every element must have finite order, but the converse is false. There are infinite groups where each element has finite order. Finally, find out the order of every element of a group in different types of the higher even, odd and prime order of group for composition.

Keywords
Order of Element of a Group, Multiplication Composition, LCM

Received: 9 March 2022, Accepted: 14 June 2022
https://doi.org/10.26554/sti.2022.7.3.333-343

1. INTRODUCTION

We propose to study the groups of the order of an element of a group, the order of the group, the integral powers of an element of a group, etc. Then discuss the order of every element in the higher even, odd and prime order of the group for composition. The group notation is o or *. We will frequently omit the symbol for the group operation, but we will also often write the operation as $\cdot$ or + when it represents multiplication or addition in a group and write 1 or 0 for the corresponding identity elements, respectively. Its addition +, multiplication $\times$ or (.) is used as binary operation. If the group operation is denoted as a multiplication, then an element $a \in G$ is said to be order n if n is the least positive integer such that $a^n = e$ or $o(a) \leq n$ i. e., if $a^n = e$ and $a^r \neq e\ \forall\ r \in N\ s.t.r<n$. The order of a is denoted by $o(a)$. If $a^n \neq e$ for any $n \in N$, then a is said to be of zero-order or infinite order (Beltrán et al., 2019). Let e is the identity element in $(G,+)$. An element $a \in G$ is said to be order n if $n \in Z^+$ such that $na= e$ or $o(a) \leq n$.i. e., if $na= e$ and ar$\neq e\ \forall\ r \in N$ s.t.$0<r<n$. The order of a is denoted by $o(a)$. If $na \neq e$ for any $n \in N$, then a is said to be of zero-order or infinite order (Gow, 2000). Then the classification of finite simple groups (see *e.i.* Gorenstein and Lyons, 1983; Gorenstein et al., 1999; Robinson, 2012) comes into play, and one has to be able to handle the three different families of simple groups with appropriate techniques. Nonetheless, the classification problem for finite groups into two problems: (i) identify the simple groups; (ii) identify the ways these simple groups may be put together to form bigger groups. Next, we discuss the extension of the associative property to products with any number of factors as we use the multiplication-related theorem of a group of different orders. As a result, we find out the order of a group of the higher even, odd, and prime order groups for multiplication composition. But here, we discuss the order of a group of higher odd, even, and prime order of groups as like as 69, 70, and 71. Then we find out the order of every element of a group in different types of the higher even, odd and prime order of group for composition (Mannan et al., 2021; Hall Jr, 1967; Brauer, 1942; McKay and Wales, 1971; Trefethen, 2019).

2. INTEGRAL POWERS OF AN ELEMENT OF A GROUP

2.1 Multiplication Composition (Moretó, 2021; Bächle, 2020)

Let $(G, .)$ be a group. Let $a \in G$ be an arbitrary element. By closure property, all the elements a, aa, aaa, $\cdots$ etc. belong to G. Since the composition in G is associative. Hence aaa, $\cdots$ to n factors is independent of the manner in which the factors are grouped.

If n is a positive integer, then define
$a^n = aaa\cdots$ to n factors
$a^n \in G$, by closure property
If e is identity in G, then we define $a^0 = e$.
If n is a negative integer, then by define $a^{-n} = (a^n)^{-1}$, where $(a^n)^{-1}$ is the inverse of a^n.
Consequently, $(a^n)^{-1} \in G$, since the inverse of every element of G belongs to G. $\therefore a^{-n} \in G$.

According to the definition

$$\begin{aligned}(a^n)^{-1} &= (aaa\cdots \text{to} \quad n \quad \text{factors})^{-1} \\ &= (a^{-1}a^{-1}a^{-1})\cdots \text{to} \quad n \quad \text{factors} \\ &= (a^{-1})^n \\ \therefore a^{-n} &= (a^n)^{-1} = (a^{-1})^n\end{aligned}$$

The following law of indices can be easily proved.

$$\begin{aligned} (a^m)^n &= a^{mn} \quad \forall \quad a \in G \quad \text{and} \quad \forall \quad m, n \in Z \\ \text{and} \quad a^m a^n &= a^{m+n} \quad \forall \quad a \in G \quad \text{and} \quad \forall \quad m, n \in Z \end{aligned}$$

Thus we defined a^n all integral values of n, positive, negative, or zero.

2.2 Addition Composition (Kurdachenko et al., 2019a; Kurdachenko et al., 2020b)

Let $(G,+)$ be a group. Let $a \in G$ be an arbitrary element.
By closure property, all the elements a, $a+a$, $a+a+a$, $\cdots$ etc. belong to G.
Since the composition in G is associative. Hence, $a+a+a\cdots$ to n factors is independent of the manner in which the factors are grouped.
If n is a positive integer, then define $na = a+a+a\cdots$ to n factors.
If e is identity in G, then we define $0.a = e$.
If n is a negative integer, then by define $(-n)a = -(na)$, where $-(na)$ is the inverse of na.
Consequently, $-(na) \in G$. $\therefore (-n)a \in G$.

According to the definition

$$\begin{aligned} -(na) &= -(a + a + a + \cdots + \text{to} \quad n \quad \text{factors}) \\ &= (-a) + (-a) + (-a) + \cdots + \text{to} \quad n \quad \text{factors} \\ &= n(-a) \\ \therefore (-n)a &= -(na) = n(-a) \end{aligned}$$

The following law of indices can be easily proved.

$$\begin{aligned} m(na) &= (mn)a \quad \forall \quad a \in G \quad \text{and} \quad \forall \quad m, n \in Z \\ \text{and} \quad (m+n)a &= ma + na \quad \forall \quad a \in G \quad \text{and} \quad \forall \quad m, n \in Z \end{aligned}$$

Thus we defined na all integral values of n, positive, negative, or zero.

3. SIGNIFICANCE OF THE ORDER OF AN ELEMENT OF A GROUP

We begin this section with the following theorem-related significance of the order of an element of a group.

3.1 Theorem (Robinson, 2012; Kurdachenko et al., 2020a)

Show that the order of every element of a finite group is finite.
Proof: Let G be a finite group with multiplication composition.
Let $a \in G$ be an arbitrary element.
Now we will prove that $o(a)$ is finite.
By closure property, all the elements $a^2 = a.a$, $a^3 = a.a.a$, $\cdots$ etc. belong to G.
i.e. $a, a^2, a^3, a^4, a^5, a^6, a^7, \cdots$ etc. belong to G.
But all these elements are not distinct since G is finite.
Let e be the identity in G, then $a^0 = e$.
Let us suppose that
$a^m = a^n$ where $m > n$.
$\Rightarrow a^m a^{-n} = a^n a^{-n} = a^0 = e$
$\Rightarrow a^{m-n} = e \Rightarrow a^p = e$, where $p = m-n > 0$, as $m > n$
Also, m and n are finite, and hence p is a finite positive integer.
Now p is a positive integer $s.t. a^p = e$.
This proves that
$o(a) \leq p =$ finite number
$i.e. o(a)$ finite number $\Rightarrow o(a)$ is finite.
Remark: The order of any element of a finite group can never exceed the order of the group.

3.2 Theorem (McCann, 2018; Moscatiello, 2020)

Show that the order of any integral power of an element of a group G is less than or equal to $a.i.e.$ $o(a^m) \leq o(a)$ $\forall$ $a \in G$ and $m \in N$.
Proof: Let $a \in G$ be an arbitrary element $s.t. o(a) = n$, where n is a natural number.

$$\text{Such that} \quad a^n = e = \text{identify of} \quad G. \tag{1}$$

Let a^m be any power of a and let $o(a^m) = p$
Now we will prove that $o(a^m) \leq o(a)$ $i.e.$ $p \leq n$
We have, $o(a) = n \Rightarrow a^n = e \Rightarrow a^{nm} = e^m = e \Rightarrow (a^m)^n = e \Rightarrow o(a^m) \leq n \Rightarrow p \leq n$
Remark: This theorem can also be expressed in the following ways.

i. The order of any integral power of an element a of a group cannot exceed the order of a.
ii. If $a \in G$ $i.e. G$ is a group, then $o(a^m) \leq o(a)$ $\forall$ $a \in G$ and $m \in N$.
iii. If G is a group and $a \in G$, then the order of any power of an element a is almost equal to the order of a.

4. RESULT AND DISCUSSION

Here, we discuss the result of an order of every element for multiplication composition in the higher even, odd and prime order of the group. As usual, we can use a composition-related theorem to evaluate the order of a group of different orders, such as order , etc., *i.e.*, whose order is not so high (Not Higher-Order Groups). As a result, we use a multiplication-related theorem to evaluate the order of a group of the higher odd, even, and prime order groups for composition. For that region, here we discuss the order of a group of the higher odd, even, and prime order of group as like as 69, 70, and 71.

4.1 The Higher Odd Order of a Group for Multiplication Composition (Cook et al., 2019; Kurdachenko et al., 2019b; Kurdachenko et al., 2020c)

Find the order of every element in the multiplication group $G= \{a, a^2, a^3, a^4, \cdots, a^{69}= e\}$

Solution:
We know that $o(a^m)= \frac{\lambda}{m}$, where λ = *l.c.m* of m and n

To determine $o(a^2)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 2 is 138 $\therefore o(a^2)= \frac{138}{2}= 69 \Rightarrow o(a^2)= 69$

To determine $o(a^3)$
The identity element of the given group is $a^{69}= e \Rightarrow o(a)= 69$
$\therefore o(a)= 69$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 3 is 69 $\therefore o(a^3)= \frac{69}{3}= 23 \Rightarrow o(a^3)= 23$

To determine $o(a^4)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 4 is 276 $\therefore o(a^4)= \frac{276}{4}= 69 \Rightarrow o(a^4)= 69$

To determine $o(a^5)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 5 is 345 $\therefore o(a^5)= \frac{345}{5}= 69 \Rightarrow o(a^5)= 69$

To determine $o(a^6)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 6 is 138 $\therefore o(a^6)= \frac{138}{6}= 23 \Rightarrow o(a^6)= 23$

To determine $o(a^7)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 7 is 483 $\therefore o(a^7)= \frac{483}{7}= 69 \Rightarrow o(a^7)= 69$

To determine $o(a^8)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 8 is 552 $\therefore o(a^8)= \frac{552}{8}= 69 \Rightarrow o(a^8)= 69$

To determine $o(a^9)$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 9 is 207 $\therefore o(a^9)= \frac{207}{9}= 23 \Rightarrow o(a^9)= 23$

To determine $o(a^{10})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 10 is 690 $\therefore o(a^{10})= \frac{690}{10}= 69 \Rightarrow o(a^{10})= 69$

To determine $o(a^{11})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 11 is 759 $\therefore o(a^{11})= \frac{759}{11}= 69 \Rightarrow o(a^{11})= 69$

To determine $o(a^{12})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 12 is 276 $\therefore o(a^{12})= \frac{276}{12}= 23 \Rightarrow o(a^{12})= 23$

To determine $o(a^{13})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 13 is 897 $\therefore o(a^{13})= \frac{897}{13}= 69 \Rightarrow o(a^{13})= 69$

To determine $o(a^{14})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 14 is 966 $\therefore o(a^{14})= \frac{966}{14}= 69 \Rightarrow o(a^{14})= 69$

To determine $o(a^{15})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 15 is 345 $\therefore o(a^{15})= \frac{345}{12}= 23 \Rightarrow o(a^{15})= 23$

To determine $o(a^{16})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 16 is 1104 $\therefore o(a^{16})= \frac{1104}{14}= 69 \Rightarrow o(a^{16})= 69$

To determine $o(a^{17})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 17 is 1173 $\therefore o(a^{17})= \frac{1173}{17}= 69 \Rightarrow o(a^{17})= 69$

To determine $o(a^{18})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 18 is 414 $\therefore o(a^{18})= \frac{414}{18}= 23 \Rightarrow o(a^{18})= 23$

To determine $o(a^{19})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 19 is 1311 $\therefore o(a^{19})= \frac{1311}{19}= 69 \Rightarrow o(a^{19})= 69$

To determine $o(a^{20})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 20 is 1380 $\therefore o(a^{20})= \frac{1280}{20}= 69 \Rightarrow o(a^{20})= 69$

To determine $o(a^{21})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 21 is 483 $\therefore o(a^{21})= \frac{483}{21}= 23 \Rightarrow o(a^{21})= 23$

To determine $o(a^{22})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 22 is 1518 $\therefore o(a^{22})= \frac{1518}{22}= 69 \Rightarrow o(a^{22})= 69$

To determine $o(a^{23})$
Here, $o(a^{69})= e$,*l.c.m* of 69 and 23 is 1587 $\therefore o(a^{23})= \frac{1587}{23}= 69 \Rightarrow o(a^{23})= 69$

To determine $o(a^{24})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 24 is 552 $\therefore o(a^{24})= \frac{552}{24} = 23 \Rightarrow o(a^{24})= 23$

To determine $o(a^{25})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 25 is 1725 $\therefore o(a^{25})= \frac{1725}{25} = 69 \Rightarrow o(a^{25})= 69$

To determine $o(a^{26})$
Here, $o(a^{69})$= $e,l.c.m$ of 70 and 26 is 910 $\therefore o(a^{26})= \frac{910}{26} = 35 \Rightarrow o(a^{26})= 35$

To determine $o(a^{27})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 27 is 621 $\therefore o(a^{27})= \frac{621}{27} = 23 \Rightarrow o(a^{27})= 23$

To determine $o(a^{28})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 28 is 1932 $\therefore o(a^{28})= \frac{1932}{28} = 69 \Rightarrow o(a^{28})= 69$

To determine $o(a^{29})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 29 is 2001 $\therefore o(a^{29})= \frac{2001}{29} = 69 \Rightarrow o(a^{29})= 69$

To determine $o(a^{30})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 30 is 690 $\therefore o(a^{30})= \frac{690}{30} = 23 \Rightarrow o(a^{30})= 23$

To determine $o(a^{31})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 31 is 2139 $\therefore o(a^{31})= \frac{2139}{31} = 69 \Rightarrow o(a^{31})= 69$

To determine $o(a^{32})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 32 is 2208 $\therefore o(a^{32})= \frac{2208}{32} = 69 \Rightarrow o(a^{32})= 69$

To determine $o(a^{33})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 33 is 759 $\therefore o(a^{33})= \frac{759}{33} = 23 \Rightarrow o(a^{32})= 23$

To determine $o(a^{34})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 34 is 2346 $\therefore o(a^{34})= \frac{2346}{34} = 69 \Rightarrow o(a^{34})= 69$

To determine $o(a^{35})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 35 is 2415 $\therefore o(a^{35})= \frac{2415}{35} = 69 \Rightarrow o(a^{35})= 69$

To determine $o(a^{36})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 36 is 828 $\therefore o(a^{36})= \frac{828}{36} = 23 \Rightarrow o(a^{36})= 23$

To determine $o(a^{37})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 37 is 2553 $\therefore o(a^{37})= \frac{2553}{37} = 69 \Rightarrow o(a^{37})= 69$

To determine $o(a^{38})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 38 is 2622 $\therefore o(a^{38})= \frac{2622}{38} = 69 \Rightarrow o(a^{38})= 69$

To determine $o(a^{39})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 39 is 897 $\therefore o(a^{39})= \frac{897}{39} = 23 \Rightarrow o(a^{39})= 23$

To determine $o(a^{40})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 40 is 2760 $\therefore o(a^{40})= \frac{2760}{40} = 69 \Rightarrow o(a^{40})= 69$

To determine $o(a^{41})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 41 is 2829 $\therefore o(a^{41})= \frac{2829}{41} = 69 \Rightarrow o(a^{41})= 69$

To determine $o(a^{42})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 42 is 966 $\therefore o(a^{42})= \frac{966}{42} = 23 \Rightarrow o(a^{42})= 23$

To determine $o(a^{43})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 43 is 2967 $\therefore o(a^{43})= \frac{2967}{43} = 69 \Rightarrow o(a^{43})= 69$

To determine $o(a^{44})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 44 is 3036 $\therefore o(a^{44})= \frac{3036}{44} = 69 \Rightarrow o(a^{44})= 69$

To determine $o(a^{45})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 45 is 1035 $\therefore o(a^{32})= \frac{1035}{45} = 23 \Rightarrow o(a^{45})= 23$

To determine $o(a^{46})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 46 is 3174 $\therefore o(a^{46})= \frac{3174}{46} = 69 \Rightarrow o(a^{46})= 69$

To determine $o(a^{47})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 32 is 3243 $\therefore o(a^{47})= \frac{3243}{47} = 69 \Rightarrow o(a^{47})= 69$

To determine $o(a^{48})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 48 is 1104 $\therefore o(a^{48})= \frac{1104}{48} = 69 \Rightarrow o(a^{48})= 69$

To determine $o(a^{49})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 49 is 3381 $\therefore o(a^{49})= \frac{2281}{49} = 69 \Rightarrow o(a^{49})= 69$

To determine $o(a^{50})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 50 is 3450 $\therefore o(a^{50})= \frac{3450}{50} = 69 \Rightarrow o(a^{50})= 69$

To determine $o(a^{25})$
Here, $o(a^{69})$= $e,l.c.m$ of 69 and 51 is 1173 $\therefore o(a^{51})= \frac{1173}{51} = 23 \Rightarrow o(a^{51})= 23$

To determine $o(a^{52})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 52 is 3588 $\therefore o(a^{52})= \frac{3588}{52}= 69$
$\Rightarrow o(a^{52})= 69$

To determine $o(a^{53})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 53 is 3657 $\therefore o(a^{53})= \frac{3657}{53}= 69$
$\Rightarrow o(a^{53})= 69$

To determine $o(a^{54})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 54 is 1242 $\therefore o(a^{54})= \frac{1242}{54}= 23$
$\Rightarrow o(a^{54})= 23$

To determine $o(a^{55})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 55 is 3795 $\therefore o(a^{55})= \frac{3795}{55}= 69$
$\Rightarrow o(a^{55})= 69$

To determine $o(a^{56})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 56 is 3864 $\therefore o(a^{56})= \frac{3864}{56}= 69$
$\Rightarrow o(a^{56})= 69$

To determine $o(a^{57})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 57 is 1311 $\therefore o(a^{57})= \frac{1311}{57}= 23$
$\Rightarrow o(a^{57})= 23$

To determine $o(a^{58})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 58 is 4002 $\therefore o(a^{58})= \frac{4002}{58}= 69$
$\Rightarrow o(a^{58})= 69$

To determine $o(a^{59})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 59 is 4071 $\therefore o(a^{59})= \frac{4071}{59}= 69$
$\Rightarrow o(a^{59})= 69$

To determine $o(a^{60})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 60 is 1380 $\therefore o(a^{60})= \frac{1380}{60}= 23$
$\Rightarrow o(a^{60})= 23$

To determine $o(a^{61})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 61 is 4209 $\therefore o(a^{61})= \frac{4209}{61}= 69$
$\Rightarrow o(a^{61})= 69$

To determine $o(a^{62})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 62 is 4278 $\therefore o(a^{62})= \frac{4278}{62}= 69$
$\Rightarrow o(a^{62})= 69$

To determine $o(a^{63})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 63 is 1449 $\therefore o(a^{63})= \frac{1449}{63}= 23$
$\Rightarrow o(a^{63})= 23$

To determine $o(a^{64})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 64 is 4416 $\therefore o(a^{64})= \frac{4416}{64}= 69$
$\Rightarrow o(a^{64})= 69$

To determine $o(a^{65})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 65 is 4485 $\therefore o(a^{65})= \frac{4485}{65}= 69$
$\Rightarrow o(a^{65})= 69$

To determine $o(a^{66})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 66 is 1518 $\therefore o(a^{66})= \frac{1518}{66}= 23$
$\Rightarrow o(a^{66})= 23$

To determine $o(a^{67})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 67 is 4623 $\therefore o(a^{67})= \frac{4623}{67}= 69$
$\Rightarrow o(a^{67})= 69$

To determine $o(a^{68})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 68 is 4692 $\therefore o(a^{68})= \frac{4692}{68}= 69$
$\Rightarrow o(a^{68})= 69$

To determine $o(a^{69})$
Here, $o(a^{69})= e$, $l.c.m$ of 69 and 69 is 69 $\therefore o(a^{69})= \frac{69}{69}= 1 \Rightarrow$
$o(a^{69})= 1$

4.2 The Higher Even Order of a Group for Multiplication Composition (Kurdachenko et al., 2020c; Russo, 2019; Subbotin, 2019)

Find the order of every element in the multiplication group $G= \{a, a^2, a^3, a^4, \cdots, a^{70}= e\}$

Solution:
The identity element of the given group is $a^{70}= e \Rightarrow o(a)= 70$
$\therefore o(a)= 70$
We know that $o(a^m)= \frac{\lambda}{m}$, where $\lambda = l.c.m$ of m and n

To determine $o(a^2)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 2 is 70 $\therefore o(a^2)= \frac{70}{2}= 35 \Rightarrow o(a^2)=$
35

To determine $o(a^3)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 3 is 210 $\therefore o(a^3)= \frac{210}{3}= 70 \Rightarrow$
$o(a^3)= 70$

To determine $o(a^4)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 4 is 140 $\therefore o(a^4)= \frac{140}{4}= 35 \Rightarrow$
$o(a^4)= 35$

To determine $o(a^5)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 5 is 70 $\therefore o(a^5)= \frac{70}{5}= 14 \Rightarrow o(a^5)=$
14

To determine $o(a^6)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 6 is 210 $\therefore o(a^6)= \frac{210}{6}= 35 \Rightarrow$
$o(a^6)= 35$

To determine $o(a^7)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 7 is 70 $\therefore o(a^7)= \frac{70}{7}= 10 \Rightarrow o(a^7)=$
10

To determine $o(a^8)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 8 is 280 $\therefore o(a^8)= \frac{280}{8}= 35 \Rightarrow$
$o(a^8)= 35$

To determine $o(a^9)$
Here, $o(a^{70})= e$, $l.c.m$ of 70 and 9 is 630 $\therefore o(a^9)= \frac{630}{9}= 70 \Rightarrow$
$o(a^9)= 70$

To determine $o(a^{10})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 10 is 70 $\therefore o(a^{10})$= $\frac{70}{10}$= 7 $\Rightarrow$ $o(a^{10})$= 7

To determine $o(a^{11})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 11 is 770 $\therefore o(a^{11})$= $\frac{770}{11}$= 70 $\Rightarrow$ $o(a^{11})$= 70

To determine $o(a^{12})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 12 is 420 $\therefore o(a^{12})$= $\frac{420}{12}$= 35 $\Rightarrow$ $o(a^{12})$= 35

To determine $o(a^{13})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 13 is 910 $\therefore o(a^{13})$= $\frac{910}{13}$= 70 $\Rightarrow$ $o(a^{13})$= 70

To determine $o(a^{14})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 14 is 70 $\therefore o(a^{14})$= $\frac{70}{14}$= 5 $\Rightarrow$ $o(a^{14})$= 5

To determine $o(a^{15})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 15 is 210 $\therefore o(a^{15})$= $\frac{210}{15}$= 14 $\Rightarrow$ $o(a^{15})$= 14

To determine $o(a^{16})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 16 is 560 $\therefore o(a^{16})$= $\frac{560}{16}$= 35 $\Rightarrow$ $o(a^{16})$= 35

To determine $o(a^{17})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 17 is 1190 $\therefore o(a^{17})$= $\frac{1190}{17}$= 70 $\Rightarrow$ $o(a^{17})$= 70

To determine $o(a^{18})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 18 is 630 $\therefore o(a^{18})$= $\frac{630}{18}$= 35 $\Rightarrow$ $o(a^{18})$= 35

To determine $o(a^{19})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 19 is 1330 $\therefore o(a^{19})$= $\frac{1330}{19}$= 70 $\Rightarrow$ $o(a^{19})$= 70

To determine $o(a^{20})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 20 is 140 $\therefore o(a^{20})$= $\frac{140}{20}$= 7 $\Rightarrow$ $o(a^{20})$= 7

To determine $o(a^{21})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 21 is 210 $\therefore o(a^{21})$= $\frac{210}{21}$= 10 $\Rightarrow$ $o(a^{21})$= 10

To determine $o(a^{22})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 22 is 770 $\therefore o(a^{22})$= $\frac{770}{22}$= 35 $\Rightarrow$ $o(a^{22})$= 35

To determine $o(a^{23})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 23 is 1610 $\therefore o(a^{23})$= $\frac{1610}{23}$= 70 $\Rightarrow$ $o(a^{23})$= 70

To determine $o(a^{24})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 24 is 840 $\therefore o(a^{24})$= $\frac{840}{24}$= 35 $\Rightarrow$ $o(a^{24})$= 35

To determine $o(a^{25})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 25 is 350 $\therefore o(a^{25})$= $\frac{350}{25}$= 14 $\Rightarrow$ $o(a^{25})$= 14

To determine $o(a^{26})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 26 is 910 $\therefore o(a^{26})$= $\frac{910}{26}$= 35 $\Rightarrow$ $o(a^{26})$= 35

To determine $o(a^{27})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 27 is 1890 $\therefore o(a^{27})$= $\frac{1890}{27}$= 70 $\Rightarrow$ $o(a^{27})$= 70

To determine $o(a^{28})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 28 is 140 $\therefore o(a^{28})$= $\frac{140}{28}$= 5 $\Rightarrow$ $o(a^{28})$= 5

To determine $o(a^{29})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 29 is 2030 $\therefore o(a^{29})$= $\frac{2030}{29}$= 70 $\Rightarrow$ $o(a^{29})$= 70

To determine $o(a^{30})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 30 is 210 $\therefore o(a^{30})$= $\frac{210}{30}$= 7 $\Rightarrow$ $o(a^{30})$= 7

To determine $o(a^{31})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 31 is 2170 $\therefore o(a^{31})$= $\frac{2170}{31}$= 70 $\Rightarrow$ $o(a^{31})$= 70

To determine $o(a^{32})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 32 is 1120 $\therefore o(a^{32})$= $\frac{1120}{32}$= 35 $\Rightarrow$ $o(a^{32})$= 35

To determine $o(a^{33})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 33 is 2310 $\therefore o(a^{33})$= $\frac{2310}{33}$= 70 $\Rightarrow$ $o(a^{33})$= 70

To determine $o(a^{34})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 34 is 1190 $\therefore o(a^{34})$= $\frac{1190}{34}$= 35 $\Rightarrow$ $o(a^{34})$= 35

To determine $o(a^{35})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 35 is 490 $\therefore o(a^{35})$= $\frac{490}{35}$= 14 $\Rightarrow$ $o(a^{35})$= 14

To determine $o(a^{36})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 36 is 1260 $\therefore o(a^{35})$= $\frac{1260}{35}$= 35 $\Rightarrow$ $o(a^{36})$= 35

To determine $o(a^{37})$
Here, $o(a^{70})$= *e,l.c.m* of 70 and 37 is 2590 $\therefore o(a^{37})$= $\frac{2590}{37}$= 70 $\Rightarrow$ $o(a^{37})$= 70

To determine $o(a^{38})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 38 is 1330 $\therefore o(a^{38})$= $\frac{1330}{38}$= 35 $\Rightarrow o(a^{38})$= 35

To determine $o(a^{39})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 39 is 2730 $\therefore o(a^{39})$= $\frac{2730}{30}$= 70 $\Rightarrow o(a^{39})$= 70

To determine $o(a^{40})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 40 is 280 $\therefore o(a^{40})$= $\frac{280}{40}$= 7 $\Rightarrow o(a^{40})$= 7

To determine $o(a^{41})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 41 is 2870 $\therefore o(a^{41})$= $\frac{2870}{41}$= 70 $\Rightarrow o(a^{41})$= 70

To determine $o(a^{42})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 42 is 1470 $\therefore o(a^{42})$= $\frac{1470}{4}$= 35 $\Rightarrow o(a^{42})$= 35

To determine $o(a^{43})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 43 is 3010 $\therefore o(a^{43})$= $\frac{3010}{43}$= 70 $\Rightarrow o(a^{43})$= 70

To determine $o(a^{44})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 44 is 1540 $\therefore o(a^{44})$= $\frac{1540}{44}$= 35 $\Rightarrow o(a^{44})$= 35

To determine $o(a^{45})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 45 is 630 $\therefore o(a^{630})$= $\frac{630}{45}$= 45 $\Rightarrow o(a^{44})$= 14

To determine $o(a^{46})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 46 is 1610 $\therefore o(a^{46})$= $\frac{1610}{46}$= 35 $\Rightarrow o(a^{46})$= 35

To determine $o(a^{47})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 47 is 3290 $\therefore o(a^{47})$= $\frac{3290}{47}$= 70 $\Rightarrow o(a^{47})$= 70

To determine $o(a^{48})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 48 is 1680 $\therefore o(a^{48})$= $\frac{1680}{48}$= 35 $\Rightarrow o(a^{48})$= 35

To determine $o(a^{49})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 49 is 490 $\therefore o(a^{49})$= $\frac{490}{49}$= 10 $\Rightarrow o(a^{49})$= 10

To determine $o(a^{50})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 50 is 350 $\therefore o(a^{50})$= $\frac{350}{50}$= 7 $\Rightarrow o(a^{50})$= 7

To determine $o(a^{51})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 51 is 3570 $\therefore o(a^{51})$= $\frac{3570}{51}$= 70 $\Rightarrow o(a^{51})$= 70

To determine $o(a^{52})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 52 is 1820 $\therefore o(a^{52})$= $\frac{1820}{52}$= 35 $\Rightarrow o(a^{52})$= 35

To determine $o(a^{53})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 53 is 3710 $\therefore o(a^{53})$= $\frac{3710}{53}$= 70 $\Rightarrow o(a^{53})$= 70

To determine $o(a^{54})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 54 is 1890 $\therefore o(a^{54})$= $\frac{1890}{54}$= 35 $\Rightarrow o(a^{54})$= 35

To determine $o(a^{55})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 55 is 770 $\therefore o(a^{55})$= $\frac{770}{55}$= 14 $\Rightarrow o(a^{55})$= 14

To determine $o(a^{56})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 56 is 280 $\therefore o(a^{56})$= $\frac{280}{56}$= 5 $\Rightarrow o(a^{56})$= 5

To determine $o(a^{57})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 57 is 3990 $\therefore o(a^{57})$= $\frac{3990}{57}$= 70 $\Rightarrow o(a^{57})$= 70

To determine $o(a^{58})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 58 is 2030 $\therefore o(a^{58})$= $\frac{2030}{58}$= 35 $\Rightarrow o(a^{58})$= 35

To determine $o(a^{59})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 59 is 4130 $\therefore o(a^{59})$= $\frac{4130}{59}$= 70 $\Rightarrow o(a^{59})$= 70

To determine $o(a^{60})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 60 is 420 $\therefore o(a^{60})$= $\frac{420}{60}$= 7 $\Rightarrow o(a^{60})$= 7

To determine $o(a^{61})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 61 is 4270 $\therefore o(a^{61})$= $\frac{4270}{61}$= 70 $\Rightarrow o(a^{61})$= 70

To determine $o(a^{62})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 62 is 2170 $\therefore o(a^{62})$= $\frac{2170}{62}$= 35 $\Rightarrow o(a^{62})$= 35

To determine $o(a^{63})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 63 is 4410 $\therefore o(a^{63})$= $\frac{4410}{63}$= 70 $\Rightarrow o(a^{63})$= 70

To determine $o(a^{64})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 64 is 2240 $\therefore o(a^{64})$= $\frac{2240}{64}$= 35 $\Rightarrow o(a^{64})$= 35

To determine $o(a^{65})$
Here, $o(a^{70})$= $e,l.c.m$ of 70 and 65 is 910 $\therefore o(a^{65})$= $\frac{910}{65}$= 14 $\Rightarrow o(a^{65})$= 14

To determine $o(a^{66})$
Here, $o(a^{70})$= e,$l.c.m$ of 70 and 66 is 2310 $\therefore o(a^{66})= \frac{2310}{66} = 35$ $\Rightarrow o(a^{66})= 35$

To determine $o(a^{67})$
Here, $o(a^{70})$= e,$l.c.m$ of 70 and 67 is 4690 $\therefore o(a^{70})= \frac{4690}{67} = 70$ $\Rightarrow o(a^{67})= 70$

To determine $o(a^{68})$
Here, $o(a^{70})$= e,$l.c.m$ of 70 and 68 is 2380 $\therefore o(a^{68})= \frac{2380}{68} = 35$ $\Rightarrow o(a^{68})= 35$

To determine $o(a^{69})$
Here, $o(a^{70})$= e,$l.c.m$ of 70 and 69 is 4830 $\therefore o(a^{69})= \frac{4830}{69} = 70$ $\Rightarrow o(a^{69})= 70$

To determine $o(a^{70})$
Here, $o(a^{70})$= e,$l.c.m$ of 70 and 70 is 70 $\therefore o(a^{70})= \frac{70}{70} = 1 \Rightarrow$ $o(a^{70})= 1$

4.3 The Higher Prime Order of a Group for Multiplication Composition (d'Angeli et al., 2020; Gillibert, 2018; Cavaleri et al., 2021)

Find the order of every element in the multiplication group $G= \{a, a^2, a^3, a^4, \cdots, a^{71} = e\}$

Solution:
The identity element of the given group is $a^{71} = e \Rightarrow o(a)= 71$
$\therefore o(a)= 71$
We know that $o(a^m)= \frac{\lambda}{m}$, where $\lambda = l.c.m$ of m and n

To determine $o(a^2)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 2 is 142 $\therefore o(a^2)= \frac{142}{2} = 71 \Rightarrow$ $o(a^2)= 71$

To determine $o(a^3)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 3 is 213 $\therefore o(a^3)= \frac{213}{3} = 71 \Rightarrow$ $o(a^3)= 71$

To determine $o(a^4)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 4 is 284 $\therefore o(a^4)= \frac{284}{4} = 71 \Rightarrow$ $o(a^4)= 71$

To determine $o(a^5)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 5 is 355 $\therefore o(a^5)= \frac{355}{5} = 71 \Rightarrow$ $o(a^5)= 71$

To determine $o(a^6)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 6 is 426 $\therefore o(a^6)= \frac{426}{6} = 71 \Rightarrow$ $o(a^6)= 71$

To determine $o(a^7)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 7 is 497 $\therefore o(a^7)= \frac{497}{7} = 71 \Rightarrow$ $o(a^7)= 71$

To determine $o(a^8)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 8 is 568 $\therefore o(a^8)= \frac{568}{8} = 71 \Rightarrow$ $o(a^8)= 71$

To determine $o(a^9)$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 9 is 639 $\therefore o(a^9)= \frac{639}{9} = 71 \Rightarrow$ $o(a^9)= 71$

To determine $o(a^{10})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 10 is 710 $\therefore o(a^{10})= \frac{710}{10} = 71 \Rightarrow$ $o(a^{10})= 71$

To determine $o(a^{11})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 11 is 781 $\therefore o(a^{11})= \frac{781}{11} = 71 \Rightarrow$ $o(a^{11})= 71$

To determine $o(a^{12})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 12 is 852 $\therefore o(a^{12})= \frac{852}{12} = 71 \Rightarrow$ $o(a^{12})= 71$

To determine $o(a^{13})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 13 is 923 $\therefore o(a^{13})= \frac{923}{13} = 71 \Rightarrow$ $o(a^{13})= 71$

To determine $o(a^{14})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 14 is 994 $\therefore o(a^{14})= \frac{994}{14} = 71 \Rightarrow$ $o(a^{14})= 71$

To determine $o(a^{15})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 15 is 1065 $\therefore o(a^{15})= \frac{1065}{15} = 71$ $\Rightarrow o(a^{15})= 71$

To determine $o(a^{16})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 16 is 1136 $\therefore o(a^{16})= \frac{1136}{16} = 71$ $\Rightarrow o(a^{16})= 71$

To determine $o(a^{17})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 17 is 1207 $\therefore o(a^{17})= \frac{1207}{17} = 71$ $\Rightarrow o(a^{17})= 71$

To determine $o(a^{18})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 18 is 1278 $\therefore o(a^{18})= \frac{1278}{18} = 71$ $\Rightarrow o(a^{18})= 71$

To determine $o(a^{19})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 19 is 1349 $\therefore o(a^{19})= \frac{1349}{19} = 71$ $\Rightarrow o(a^{19})= 71$

To determine $o(a^{20})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 20 is 1420 $\therefore o(a^{20})= \frac{1420}{20} = 71$ $\Rightarrow o(a^{20})= 71$

To determine $o(a^{21})$
Here, $o(a^{71})$= e,$l.c.m$ of 71 and 21 is 1491 $\therefore o(a^{21})= \frac{1491}{21} = 71$ $\Rightarrow o(a^{21})= 71$

To determine $o(a^{22})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 22 is 1562 $\therefore o(a^{22})= \frac{1562}{22} = 71$
$\Rightarrow o(a^{22}) = 71$

To determine $o(a^{23})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 23 is 1633 $\therefore o(a^{23})= \frac{1633}{23} = 71$
$\Rightarrow o(a^{23}) = 71$

To determine $o(a^{24})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 24 is 1704 $\therefore o(a^{24})= \frac{1704}{24} = 71$
$\Rightarrow o(a^{24}) = 71$

To determine $o(a^{25})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 25 is 1775 $\therefore o(a^{25})= \frac{1775}{25} = 71$
$\Rightarrow o(a^{25}) = 71$

To determine $o(a^{26})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 26 is 1846 $\therefore o(a^{26})= \frac{1846}{26} = 71$
$\Rightarrow o(a^{26}) = 71$

To determine $o(a^{27})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 27 is 1917 $\therefore o(a^{27})= \frac{1917}{27} = 71$
$\Rightarrow o(a^{27}) = 71$

To determine $o(a^{28})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 28 is 1988 $\therefore o(a^{28})= \frac{1988}{28} = 71$
$\Rightarrow o(a^{28}) = 71$

To determine $o(a^{29})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 29 is 2059 $\therefore o(a^{29})= \frac{2059}{29} = 71$
$\Rightarrow o(a^{29}) = 71$

To determine $o(a^{30})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 30 is 2130 $\therefore o(a^{30})= \frac{2130}{30} = 71$
$\Rightarrow o(a^{30}) = 71$

To determine $o(a^{31})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 31 is 2201 $\therefore o(a^{31})= \frac{2201}{31} = 71$
$\Rightarrow o(a^{31}) = 71$

To determine $o(a^{32})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 32 is 2272 $\therefore o(a^{32})= \frac{2272}{32} = 71$
$\Rightarrow o(a^{32}) = 71$

To determine $o(a^{33})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 33 is 2343 $\therefore o(a^{33})= \frac{2343}{33} = 71$
$\Rightarrow o(a^{33}) = 71$

To determine $o(a^{34})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 34 is 2414 $\therefore o(a^{34})= \frac{2414}{34} = 71$
$\Rightarrow o(a^{34}) = 71$

To determine $o(a^{35})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 35 is 2485 $\therefore o(a^{35})= \frac{2485}{35} = 71$
$\Rightarrow o(a^{35}) = 71$

To determine $o(a^{36})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 36 is 2556 $\therefore o(a^{36})= \frac{2556}{36} = 71$
$\Rightarrow o(a^{36}) = 71$

To determine $o(a^{37})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 37 is 2627 $\therefore o(a^{37})= \frac{2627}{37} = 71$
$\Rightarrow o(a^{37}) = 71$

To determine $o(a^{38})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 38 is 2698 $\therefore o(a^{38})= \frac{2698}{38} = 71$
$\Rightarrow o(a^{38}) = 71$

To determine $o(a^{39})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 39 is 2769 $\therefore o(a^{39})= \frac{2769}{39} = 71$
$\Rightarrow o(a^{39}) = 71$

To determine $o(a^{40})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 40 is 2840 $\therefore o(a^{40})= \frac{2840}{40} = 71$
$\Rightarrow o(a^{40}) = 71$

To determine $o(a^{41})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 41 is 2911 $\therefore o(a^{41})= \frac{2911}{41} = 71$
$\Rightarrow o(a^{41}) = 71$

To determine $o(a^{42})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 42 is 2982 $\therefore o(a^{42})= \frac{2982}{42} = 71$
$\Rightarrow o(a^{42}) = 71$

To determine $o(a^{43})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 43 is 3053 $\therefore o(a^{43})= \frac{3053}{43} = 71$
$\Rightarrow o(a^{43}) = 71$

To determine $o(a^{44})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 44 is 3124 $\therefore o(a^{44})= \frac{3124}{44} = 71$
$\Rightarrow o(a^{44}) = 71$

To determine $o(a^{45})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 45 is 3195 $\therefore o(a^{45})= \frac{3195}{45} = 71$
$\Rightarrow o(a^{45}) = 71$

To determine $o(a^{46})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 46 is 3266 $\therefore o(a^{46})= \frac{3266}{46} = 71$
$\Rightarrow o(a^{46}) = 71$

To determine $o(a^{47})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 47 is 3337 $\therefore o(a^{47})= \frac{3337}{47} = 71$
$\Rightarrow o(a^{47}) = 71$

To determine $o(a^{48})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 48 is 3408 $\therefore o(a^{48})= \frac{3408}{48} = 71$
$\Rightarrow o(a^{48}) = 71$

To determine $o(a^{49})$
Here, $o(a^{71})$= $e,l.c.m$ of 71 and 49 is 3479 $\therefore o(a^{49})= \frac{3479}{49} = 71$
$\Rightarrow o(a^{49}) = 71$

To determine $o(a^{50})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 50 is 3550 $\therefore o(a^{50}) = \frac{3550}{50} = 71$
$\Rightarrow o(a^{50}) = 71$

To determine $o(a^{51})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 51 is 3621 $\therefore o(a^{51}) = \frac{3621}{51} = 71$
$\Rightarrow o(a^{51}) = 71$

To determine $o(a^{52})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 52 is 3692 $\therefore o(a^{52}) = \frac{3692}{52} = 71$
$\Rightarrow o(a^{52}) = 71$

To determine $o(a^{53})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 53 is 3763 $\therefore o(a^{53}) = \frac{3763}{53} = 71$
$\Rightarrow o(a^{53}) = 71$

To determine $o(a^{54})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 54 is 3834 $\therefore o(a^{54}) = \frac{3834}{54} = 71$
$\Rightarrow o(a^{54}) = 71$

To determine $o(a^{55})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 55 is 3905 $\therefore o(a^{55}) = \frac{3905}{55} = 71$
$\Rightarrow o(a^{55}) = 71$

To determine $o(a^{56})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 56 is 3976 $\therefore o(a^{56}) = \frac{3976}{56} = 71$
$\Rightarrow o(a^{56}) = 71$

To determine $o(a^{57})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 57 is 4047 $\therefore o(a^{57}) = \frac{4047}{57} = 71$
$\Rightarrow o(a^{57}) = 71$

To determine $o(a^{58})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 58 is 4118 $\therefore o(a^{58}) = \frac{4118}{58} = 71$
$\Rightarrow o(a^{58}) = 71$

To determine $o(a^{59})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 59 is 4189 $\therefore o(a^{59}) = \frac{4189}{59} = 71$
$\Rightarrow o(a^{59}) = 71$

To determine $o(a^{60})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 60 is 4260 $\therefore o(a^{60}) = \frac{4260}{60} = 71$
$\Rightarrow o(a^{60}) = 71$

To determine $o(a^{61})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 61 is 4331 $\therefore o(a^{61}) = \frac{4331}{61} = 71$
$\Rightarrow o(a^{61}) = 71$

To determine $o(a^{62})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 62 is 4402 $\therefore o(a^{62}) = \frac{4402}{62} = 71$
$\Rightarrow o(a^{62}) = 71$

To determine $o(a^{63})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 63 is 4473 $\therefore o(a^{63}) = \frac{4473}{63} = 71$
$\Rightarrow o(a^{63}) = 71$

To determine $o(a^{64})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 64 is 4544 $\therefore o(a^{64}) = \frac{4544}{64} = 71$
$\Rightarrow o(a^{64}) = 71$

To determine $o(a^{65})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 65 is 4615 $\therefore o(a^{65}) = \frac{4615}{65} = 71$
$\Rightarrow o(a^{65}) = 71$

To determine $o(a^{66})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 66 is 4686 $\therefore o(a^{66}) = \frac{4686}{66} = 71$
$\Rightarrow o(a^{66}) = 71$

To determine $o(a^{67})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 67 is 4757 $\therefore o(a^{67}) = \frac{4757}{67} = 71$
$\Rightarrow o(a^{67}) = 71$

To determine $o(a^{68})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 68 is 4828 $\therefore o(a^{68}) = \frac{4828}{68} = 71$
$\Rightarrow o(a^{68}) = 71$

To determine $o(a^{69})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 69 is 4830 $\therefore o(a^{69}) = \frac{4899}{69} = 71$
$\Rightarrow o(a^{69}) = 71$

To determine $o(a^{70})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 70 is 4970 $\therefore o(a^{70}) = \frac{4970}{70} = 71$
$\Rightarrow o(a^{70}) = 71$

To determine $o(a^{71})$
Here, $o(a^{71})$= *e,l.c.m* of 71 and 71 is 71 $\therefore o(a^{71}) = \frac{71}{71} = 1 \Rightarrow$ $o(a^{71}) = 1$

5. CONCLUSIONS

We hope that this work will be useful for group theory related to the higher order of an element of a group. Our result is the order of every element of a group in different types of the higher-order group. This result has found extensive use in statistics, information theory and geometrics, etc. Then all expected results in this paper will help us to understand a better solution to complicated order.

6. ACKNOWLEDGMENT

I would like to thank my respectable teacher Prof. Dr. Nurul Alam Khan, for guidance throughout the research process.

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