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Research Paper

Revolutionizing Multi-Criteria Decision Making with the Triangular Fuzzy Geometry Bonferroni Mean Operator (TFGBM)

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Abstract

This study investigates the Multi-Criteria Decision Making (MCDM) topic to address the complexities of decision processes involving ambiguous information. We introduce the Triangular Fuzzy Geometric Bonferroni Mean (TFGBM) operator, a novel aggregation technique inspired by the Geometric Bonferroni Mean (GBM) concept. This operator is intended to aggregate triangular fuzzy numbers within MCDM problems effectively. We thoroughly investigate the properties of TFGBM and its distinct forms to ensure its practical utility. We introduce the Triangular Fuzzy Geometric Weighted Bonferroni Mean (TFGWBM) operator to accommodate situations where input factors have variable degrees of significance. Based on this foundation, we present a comprehensive framework for decision-making involving multiple attributes in ambiguous triangular fuzzy environments. A relevant case study regarding selecting an optimal location for a Halal center demonstrates the efficacy and applicability of our methodology. We emphasize the tangibility and efficiency of the suggested methodology in improving decision-making processes by emphasizing this real-world application.

Keywords

Aggregating Operator, Bonferroni Mean (BM), Triangular Fuzzy Number (TFN), Geometric Bonferroni Mean (GBM), Triangular Fuzzy Geometric Bonferroni Mean (TFGBM), Multi-Criteria Decision Making (MCDM)

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1. INTRODUCTION

The geometric mean (GM) is a fundamental operation among the aggregation operators. Information aggregation operators are a fascinating study area that is attracting more attention. One of the fundamental aggregation methods is the geometric mean. GM has unique characteristics and has been greatly expanded. The Bonferroni Mean (BM), originally introduced as an aggregation operator, functions predominantly as a mechanism for consolidating diverse sets of values or datasets into a singular value. This consolidation occurs within the spectrum defined by the maximal and minimal operators, alongside the logical operators "or" and "and" (Bonferroni, 1950). The precise characterization and computation of the Bonferroni mean are contingent upon the specific contextual application. Yang et al. (2023) elucidates the development of novel aggregation operators founded upon picture fuzzy soft sets, their practical implementation in the domain of medical diagnosis, and their empirical superiority substantiated through a comparative evaluative framework.

In contrast, Chakraborty and Saha (2023) introduce the Fermatean Fuzzy Multi-Criteria Group Decision Making (FFM-CGDM) technique, which leverages the BM and weighted Bonferroni mean operators to navigate uncertainties inherent in group decision-making scenarios. This approach is appropriate to identify optimal healthcare waste treatment technologies that are conducive to sustainable environmental advancement. Liang et al. (2019) offer an extension of the Bonferroni mean (EBM) within the dual hesitant environment, while Mailagaha Kumbure et al. (2020) propound a novel methodology known as the BM based fuzzy k-nearest neighbor (BM-FKNN) classifier.

Fuzzy set theory is a mathematical framework that offers a flexible and robust method to deal with uncertainty, imprecision, and vagueness in real-world problems (Benitez et al., 2007). An object can belong to a set to different degrees in fuzzy set theory, which is represented by a membership grade. Depending on the level of membership, it can range from 0 to 1, with 0 denoting no membership (Herrera-Viedma, 2015). TFNs are a type of fuzzy set that is commonly used in decision-making and controlling systems. TFNs are defined by three parameters: the minimum (s_l), most likely (s_m), and maximum value (s_r). These parameters determine the triangular shape of the TFN.

The research by Debnath (2021) explores the application of the FHSS theory, which combines fuzzy set and hypersoft set methodologies. This innovative approach addresses intricate decision-making scenarios with imprecise concepts and intricate parameter data, and the paper illustrates its fundamental set-theoretic operations through a pertinent example focused on enhancing reliability and authenticity. Another research by Cao and Štěpnička (2023) examined how to maintain the fundamental characteristics of repositioned lattice structures when using extended algebras for partial fuzzy set theory and partial fuzzy logic.

The Triangular Fuzzy Bonferroni Mean (TFBM) is a new concept that extends the traditional BM to the fuzzy domain (Zhu et al., 2015). The TFBM is used to aggregate multiple TFNs in a way that considers their membership grades. There are numerous applications for TFNs and TFBM in decision-making. In decision-making, for instance, TFBM can be used to aggregate several TFNs that represent the preferences and preferences of the decision-makers. The TFBM produced can subsequently be utilized to make better informed decisions. The GBM is a generalization of the BM that is used to combine multiple criteria into a single value by taking the geometric mean of the criteria values (Xia et al., 2013). Up until this point, there have been methods developed for the GBM. GBM can display the correlations between the gathered arguments.

The GBM is used in a variety of real-world fields, such as ecology, genomics, and medical research. Cantillo et al. (2021) apply a method centered on a modified consistent fuzzy preference relation (CFPR) that uses GBM operators in the field of fisheries and aquaculture products (FAPs). Furthermore, The GBM has the potential to be utilized in numerous other applications within the field of MCDM (Eltarabishi et al., 2020; Ismail et al., 2023a; Jin, 2023; Priyanka et al., 2023; Rodzi bin Md et al., 2023; Rodzi et al., 2021a; Wu et al., 2023). Additionally, its relevance extends to various extensions of fuzzy and neutrosophic sets (Ismail et al., 2023b; Rodzi and Ahmad, 2020; Rodzi et al., 2021b).

To address this objective, we extend the application of the GBM framework in this research by incorporating triangular fuzzy scenarios. We introduce the TFGBM operator as a novel approach to merging triangular fuzzy data, explaining its attributes and prerequisites. Furthermore, we devise the Triangle Fuzzy Geometric Weighted Bonferroni Mean (TFGWBM) operators, which lay the groundwork for making decisions involving multiple attributes within triangular fuzzy environments, especially when input factors possess varying degrees of significance.

Section 3 proposes the TFGBM operator after assessing the desired properties of the offered operators. These suggested operators are distinctive in considering the connections between input arguments. Section 4 develops the novel TFGBM operator, the GBM operator in triangular fuzzy environments. To demonstrate the advantageous qualities of the TFGBM operator, we define it and prove a theory. In Section 5, we will propose a decision-making approach for addressing real-world problems using triangular fuzzy information and the TFGWBM operator. In Section 6, we give a real-world case, selection of a suitable location for a Halal center based on multiple criteria to demonstrate the technique's viability and effectiveness. Section 7 serves as the conclusion and includes some findings.

2. EXPERIMENTAL SECTION

2.1 Preliminaries

The introductory section establishes the concepts of the paper. TFNs and their characterizing conditions have been defined. The operational principles and degree of possibility of TFNs are explained. The BM and GBM operators are essential for aggregation and have been defined in this section.

Definition 1. Li et al., (2023) Let $[\tilde{s} = [s_l, s_m, s_r]]$. A fuzzy number X or M is said to be a TFN if its membership function $\mu_{\tilde{x}}$:M→[0,1] has the following conditions:

$$\mu_{\tilde{x}}(x)=\begin{bmatrix} \frac{x-s_l}{s_m-s_l}, for s_l \le x \le s_m \\ 1, for x = s_m \\ \frac{s_r-x}{s_r-s_m}, for s_m \le x \le s_r \\ 0, otherwise \end{bmatrix} \quad (1)$$

Definition 2. Liu et al. (2023) The fundamental operational rules for TFNs, let s=[s_l,s_m,s_r] and t=[t_l, t_m, t_r] be:

$$s \oplus t = [s_l, s_m, s_r] \oplus [t_l, t_m, t_r] = [s_l + t_l, s_m + t_m, s_r + t_r] \quad (2)$$

$$s \oplus t = [s_l, s_m, s_r] \oplus [t_l, t_m, t_r] = [s_l t_l, s_m t_m, s_r t_r] \quad (3)$$

$$T \oplus s = T \oplus [s_l, s_m, s_r] = [T s_l, T s_m, T s_r], T > 0. \quad (4)$$

Definition 3. Liu et al. (2023) The degree of possibility s≤ t is defined as s=[s_l,s_m,s_r] if and t=[t_l,t_m,t_r] are triangular fuzzy integers.

$$\rho(s \ge t) = \lambda max \left\{1 - max\left[\frac{t_m - s_l}{s_m - s_l + t_m t_l}\right], 0\right\} + (1-\lambda)$$
$$max\left\{1 - max\left[\frac{t_r - s_m}{s_r - s_m + t_r - t_m}\right], 0\right\} \quad (5)$$

where the value serves as an indicator of the individual's attitude towards risk, reflecting their propensity to take on risks. The decision maker demonstrates risk-seeking behavior if λ>0.5, whereas $\lambda = 0.5$ signifies a neutral stance towards risk. On the other hand, if $\lambda < 0.5$ the decision maker is characterised as being averse to risk. The following can be deduced from Definition 3:

1)$0 \leq \rho(s \geq t) \leq 1, 0 \leq \rho(t \geq s) \leq 1$;
2)$\rho(s \geq t) + \rho(t \geq s) = 1$

Especially,

$$\rho(s \geq t) = \rho(t \geq s) = 0.5$$

Definition 4. Liang et al. (2019) A fuzzy number is a fuzzy set specified by A=x,μA(x) with membership function μ_A and x taking its number on the real line, and it has the following attributes:

1.A continuous mapping to the closed interval $[0, 1]$.
2.Constant on $(-\infty, s] : \mu_A(x) = 0 \forall x \in (-\infty, s]$.
3.Strictly increasing on $[s, t]$.
4.Constant on $[t, u] : \mu_A(x) = 1 \forall x \in [t, u]$.
5.Strictly decreasing on $[u, d]$.
6.Constant on $[d, \infty) : \mu_A(x) = 0 \forall x \in [d, \infty]$.

Definition 5. Liu et al. (2023) Let s_i (i=1,2,...,n) and p, q≥ 0 represent a set of non-negative real integers. Then:

$$BM^{p,q}(s_l, s_2, \dots, s_n) = \left[\frac{1}{n(n-1)} \sum_{i,j=1 i \neq j}^{n} s_i^p s_j^q \right]^{\frac{1}{p+q}} \quad (6)$$

Definition 6. Chen et al. (2019) Let Let s_i (i=1,2,...,n) and p, q≥ represent a set of non-negative real integers. Then:

$$GB^{p,q}(s_l, s_2, \dots, s_n) = \frac{1}{p+q} \prod_{i,j=1 i \neq j}^{n} (ps_i + qs_j)^{\frac{1}{n(n-1)}} \quad (7)$$

The Equation (7) is called Geometric Bonferroni Mean (GBM). Some of the properties of the GBM include:

1.$GB^{p,q}(0, 0, \dots, 0) = 0$.
2.$GB^{p,q}(s, s, \dots, s) = a$ if $s_i = s$ for all i.
3.$GB^{p,q}(s_l, s_2, \dots, s_n) \geq GB^{p,q}(d_l, d_2, \dots, d_n)$,
$i.e., GB^{p,q}$ is monotonic , if $s_i \geq d_i$, for all i.
4. $\min_i s \leq GB^{p,q}(s_l, s_2, \dots, s_n) \leq \max_i s_i$.

3. RESULTS AND DISCUSSION

3.1 The Novel Triangular Fuzzy Geometric Bonferroni Mean

We introduce the novel TFGBM operator in this section. We present the TFGBM as an extension of the GBM tailored for triangular fuzzy contexts based on previously discussed concepts. This operator may improve MCDM in ambiguous situations involving TFN. We define the TFGBM operator and elaborate on its fundamental properties and theoretical foundations. This investigation provides insights into the operational capacities of the operator and their consequential influence on decision-making within real-world contexts.

Definition 6. Let $\tilde{s}_i$=(s_i^l,s_i^m,s_i^r)(i=1,2,...,n) be a set of TFNs and let p,q>0. If

$$TFGBM^{p,q}(\tilde{s}_l, \tilde{s}_2, \dots, \tilde{s}_n) = \frac{1}{p+q} \prod_{i,j=1 i \neq j}^{n} (p\tilde{s}_i + q\tilde{s}_j)^{\frac{1}{n(n-1)}}$$

$$= \left[\left(\frac{1}{p+q} \prod_{i,j=1 i \neq j}^{n} (p\tilde{s}_i^l + q\tilde{s}_j^l)^{\frac{1}{n(n-1)}} \right), \right.$$
$$= \left(\frac{1}{p+q} \prod_{i,j=1 i \neq j}^{n} (p\tilde{s}_i^m + q\tilde{s}_j^m)^{\frac{1}{n(n-1)}} \right),$$
$$\left. = \left(\frac{1}{p+q} \prod_{i,j=1 i \neq j}^{n} (p\tilde{s}_i^r + q\tilde{s}_j^r)^{\frac{1}{n(n-1)}} \right) \right] \quad (8)$$

The Equation (8) is called the TFGBM operator. Some of the properties of the TFGBM include:

Theorem 1. Idempotency: Let $\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)(i=1,2,...,n) it be a set of TFNs. If all $\tilde{s}_j$ [$\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)] are equal, i.e., $\tilde{s}_j$ [$\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)]=$\tilde{s}$[$\tilde{s}$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)] for all j, then. TFGBM^{p,q}($\tilde{s}_1$,$\tilde{s}_2$, ...,$\tilde{s}_n$)=$\tilde{s}$.

Theorem 2. Boundedness: Let $\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)(i=1,2,...,n) be a set of TFNs and let $\tilde{s}^-$ $\min_j \tilde{s}_j$, $\tilde{s}^+$= $\tilde{s}^-$ $\min_j \tilde{s}_j$ $\tilde{s}^- \leq$ TFGBM ^{p,q} ($\tilde{s}_i$,$\tilde{s}_2$,...,$\tilde{s}_n$) $\leq \tilde{s}^+$ them.

Theorem 3. Monotonicity: Let $\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)(i=1,2,...,n) and $\tilde{s}'_i$ ($s'^l{}_i$,$s'^m{}_i$,$s'^r{}_i$) (i=1,2,...,n) be two sets of TFNs, if $\tilde{s}_j \leq \tilde{s}'_j$ for all, then TFGBM ^{p,q} ($\tilde{s}_i$,$\tilde{s}_2$,...,$\tilde{s}_n$) ≤ TFGBM^{p,q} ($\tilde{s}'_i$,$\tilde{s}'_2$,...,$\tilde{s}'_n$).

Theorem 4. Commutativity: Let $\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$) and $\tilde{s}'_i$ ($s'^l{}_i$,$s'^m{}_i$,$s'^r{}_i$) (i=1,2,...,n) and be two sets of TFNs, where $\tilde{s}'_i$= ($s'^l{}_i$,$s'^m{}_i$,$s'^r{}_i$)(i=1,2,...,n) is any permutation of $\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$) (i=1,2,...,n), then TFGBM^{p,q} ($\tilde{s}_i$,$\tilde{s}_2$,...,$\tilde{s}_n$) = TFGBM^{p,q} ($\tilde{s}'_i$,$\tilde{s}'_2$,..., $\tilde{s}'_n$).

Case 1. To simplify, we employ the triangular fuzzy geometric mean (TFGM) operator if p=1 and q→ 0, then by the Equation (8):

$$TFGBM^{1,0}(\tilde{s}_i, \tilde{s}_2, \dots, \tilde{s}_n) = \prod_{i=1}^{n} (\tilde{s}_i)^{\frac{1}{n(n-1)}} \quad (9)$$

Case 2. When p=1 and q=1, we obtain the triangular fuzzy geometric interrelated square mean (TFGISM) operator, which is represented by Equation (8):

$$TFGBM^{1,1}(\tilde{s}_i, \tilde{s}_2, ..., \tilde{s}_n) = \frac{1}{2}\prod_{i,j=1 i\neq j}^{n}(\tilde{s}_i + \tilde{s}_j)^{\frac{1}{n(n-1)}} \quad (10)$$

Case 3. When p=2 and q → 0, we have the triangular fuzzy geometric square mean (TFGSM) operator, which is represented by Equation (8):

$$TFGBM^{2,0}(\tilde{s}_i, \tilde{s}_2, ..., \tilde{s}_n) = \frac{1}{2}\prod_{i=1}^{n}(2\tilde{s}_i)^{\frac{1}{n(n-1)}} \quad (11)$$

Case 4. When q → 0, we get the triangular fuzzy geometric generalized mean (TFGGM) operator if q → 0, which is represented by Equation (8):

$$\lim_{q\to 0} TFGBM^{p,q}(\tilde{s}_i, \tilde{s}_2, ..., \tilde{s}_n) = \lim_{q\to 0}\left[\frac{1}{p+q}\prod_{i,j=1 i\neq j}^{n} (p\tilde{s}_i + q\tilde{s}_j)^{\frac{1}{n(n-1)}}\right] = \frac{1}{p}\prod_{i=1}^{n}(p\tilde{s}_i)^{\frac{1}{n(n-1)}} = TFGBM^{p,0}(\tilde{s}_i, \tilde{s}_2, ..., \tilde{s}_n) \quad (12)$$

We create the TFGWBM set of rules as follows:

Definition 7.Let $\tilde{s}_i$=($s^l{}_i$,$s^m{}_i$,$s^r{}_i$)(i=1,2,...,n) represent a set of TFNs, and p,q > 0 w=$(w_1,w_2,...,w_n)^T$ represent $\tilde{s}_i$= ($s^l{}_i$,$s^m{}_i$,$s^r{}_i$) (i=1,2,...,n) weight vector, w_i denoting the degree to which $\tilde{s}_i$, satisfying $\sum_{i=1}^{n}$ w_i=1, and w_i > (i=1,2,...,n). if

$$TFGWBM_w^{p,q}(\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n) = \left[\frac{1}{p+q}\prod_{i,j=1 i\neq j}^{n} (w_i p\tilde{s}_i + w_j q\tilde{s}_j)^{\frac{1}{n(n-1)}}\right] \quad (13)$$

3.2 Enhancing Multi-Criteria Decision Making with a Triangular Fuzzy-Based Approach

The TFGWBM operator will then be utilized to develop a method for resolving multiple criteria decision-making issues with triangle fuzzy information. Let H= (H_1,H_2,...,H_m) be a collection of alternatives and Q= (Q_1,Q_2,...,Q_m) be a collection of criteria with weight vectors of ω= (ω_1,ω_2,...,ω_M) and $\omega_j \geq 0$, j= 1,2,...,n. $\sum_{j-1}^{n}$ ω_j= 1 for a multiple criteria decision-making issue with triangular fuzzy information. Assume that H = $(\tilde{s}_{ij})$=[$s^l{}_{ij}$, $s^m{}_{ij}$, $s^r{}_{ij}]_{mxn}$ is the decision-making matrix that $\tilde{s}_{ij}$ represents the preference values provided by the decision-makers in the shape of TFNs for the alternative $H_i \in$ H concerning the criteria $Q_j \in$ Q. The following is a summary of this method:

Step 1. Using the following methods, each element in the matrix J = $(\tilde{r}_{ij})_{mxn}$ = [$\tilde{r}^l{}_{ij}$, $\tilde{r}^m{}_{ij}$, $\tilde{r}^r{}_{ij}$] that corresponded to a criteria value $\tilde{s}_{ij}$ in the matrix H was normalized: making use of the formulas below: For benefit criteria:

$$r_{ij}^l = s_{ij}^l/\sum_{i=1}^{m}\tilde{s}_{ij}^r,\ r_{ij}^m = s_{ij}^m/\sum_{i=1}^{m}s_{ij}^r,\ r_{ij}^r = s_{ij}^r/\sum_{i=1}^{m}s_{ij}^l,\ i = 1, 2, ..., m,\ j = 1, 2, ..., n, \quad (14)$$

For cost criteria:

$$r_{ij}^l = (1/s_{ij}^r)/\sum_{i=1}^{m}(1/s_{ij}^l),\ /r_{ij}^m = (1/s_{ij}^m)/\sum_{i=1}^{m}(1/s_{ij}^m),\ /r_{ij}^r = (1/s_{ij}^r)/\sum_{i=1}^{m}(1/s_{ij}^r),\ i = 1, 2, ..., m,\ j = 1, 2, ..., n. \quad (15)$$

Step 2. Apply the TFGWBM operator to the decision information in matrix J,

$$\tilde{r}_i = (r_{ij}^l, r_{ij}^m, r_{ij}^r) = TFGWBM_w^{p,q}(\tilde{r}_{i1}, \tilde{r}_{i2}, ..., \tilde{r}_{in}) = \left[\frac{1}{p+q}\prod_{k,l=1 k\neq j}^{n}(w_k p\tilde{s}_{ik} + w_l q\tilde{s}_{il})^{\frac{1}{n(n-1)}}\right],\ i = 1, 2, ..., m,\ j = 1, 2, ..., n. \quad (16)$$

Step 3. We first use Equation (5) to compare each $\tilde{r}_i$ with every $\tilde{r}_i$ (j=1,2,...,m) and then rank the overall performance values $\tilde{r}_i$ (i=1,2,...,m) To keep things simple, we let ρ_{ij} = $\rho(\tilde{r}_i \geq \tilde{r}_j)$ and then construct a complimentary matrix p=$(\rho_{ij})_{mxm}$ in which:

$$\rho_{ij} \geq 0,\ \rho_{ij} + \rho_{ji} = 1,\ \rho_{ii} = 0.5,\ i, j = 1, 2, ..., n.$$

Upon summarizing the elements within each row of matrix P, the ensuing result is as follows:

$$\rho_i = \sum_{j=1}^{m}\rho_{ij},\ i = 1, 2, ..., m,\ j = 1, 2, ..., n. \quad (17)$$

Then, following the values ρ_i(i=1,2,...,m), we rank the overall performance values $\tilde{r}_i$(i=1,2,...,m) in descending order.

Step 4. Sort all the options X_i (i-1,2,...m), and then choose the best option(s) based on the overall performance values $\tilde{r}_i$(i=1,2,...,m).

Table 1. The Illustrative Decision Matrix H

	Q_1	Q_2	Q_3	Q_4
H_1	(0.552,0.434,0.529)	(0.788,0.865,0.312)	(0.947,0.453,0.450)	(0.216,0.342,0.776)
H_2	(0.679,0.536,0.287)	(0.341,0.490,0.635)	(0.529,0.035,0.824)	(0.440,0.810,0.486)
H_3	(0.239,0.777,0.972)	(0.098,0.366,0.776)	(0.370,0.810,0.210)	(0.588,0.862,0.599)
H_4	(0.725,0.107,0.268)	(0.816,0.378,0.442)	(0.490,0.490,0.909)	(0.364,0.554,0.534)
H_5	(0.082,0.360,0.050)	(0.163,0.552,0.756)	(0.072,0.668,0.040)	(0.678,0.704,0.512)

Table 2. The Normalized Decision Matrix J

	Q_1	Q_2	Q_3	Q_4
H_1	(0.262,0.196,0.233)	(0.270,0.326,0.142)	(0.103,0.061,0.031)	(0.074,0.105,0.339)
H_2	(0.322,0.242,0.126)	(0.117,0.185,0.288)	(0.056,0.806,0.055)	(0.151,0.247,0.212)
H_3	(0.113,0.351,0.427)	(0.033,0.138,0.352)	(0.221,0.034,0.079)	(0.202,0.263,0.262)
H_4	(0.345,0.048,0.118)	(0.279,0.143,0.200)	(0.051,0.057,0.060)	(0.125,0.169,0.234)
H_5	(0.039,0.163,0.022)	(0.056,0.208,0.343)	(1.156,0.042,0.405)	(0.233,0.215,0.224)

Table 3. The Rankings of the Cities

	TFGWBM	Rank
H_1	(0.035,0.033,0.041)	3
H_2	(0.035,0.088,0.036)	4
H_3	(0.038,0.043,0.059)	1
H_4	(0.039,0.024,0.034)	5
H_5	(0.070,0.034,0.055)	2

3.3 Numerical Example

We provide a concrete instance of our methodology as applied to the task of identifying the most suitable site for a Halal center from a selection of five distinct cities. Our objective is to determine the most appropriate city in which to establish a prosperous Halal centre. To facilitate this selection, we evaluate each city's potential as a Halal centre based on three benefit criteria, (β):(Q_1) Accessibility, (Q_2) Halal Infrastructure, and (Q_4) Market Demand, in addition to one cost criterion, (X):(Q_3) Regulatory obstacles.

The weighting vector for four criteria ω = (0.2, 0.1, 0.3, 0.4) is, used by the decision-makers to assess the five potential cities H_i (i=1,2,...,5) which will construct the matrix H=$(\tilde{s}_{ij})_{5x4}$ that can be seen in Table 1.

The values that the TFGWBM operator returns are determined by the parameters p and q, and it is clear from Equation (16) and the example that these two parameters are unreliable. The TFGWBM operator is unable to comprehend the interrelationship between the individual arguments in the uncommon circumstance that at least one of the two parameters has a value of zero. The calculation work needed usually increases with the size of the parameters p and q. This leads to the common assumption in real-world applications that the two parameters have values of p =1 and q=1, which is clear-cut and understandable and enables full consideration of the interrelationship between the different arguments.

Step 1. Normalize the decision matrix J, whose values are shown in Table 2.

Step 2. The city's overall triangular fuzzy performance value $\tilde{r}_i$(i=1,2,3,4,5) for H_i is obtained by aggregating all the $\tilde{r}_{ij}$(j=1,2,...,n) triangular fuzzy performance values using the TFGWBM operator. The results of this aggregation process are presented in Table 3.

Step 3. Table 3 presents the rankings of the cities according to the findings aggregated in Table 3 and the formula used to calculate the degree of possibility using Equation (5). As we can see, the option H_3 is the best alternatives.

4. CONCLUSION

This work has presented the TFGBM operator within the field of MCDM. The incorporation of TFNs into the GBM operator provides a valuable means of addressing the challenges posed by uncertainty and complexity in decision-making processes. The examination of its characteristics, together with a pragmatic case study pertaining to the identification of a suitable halal hub centre, highlights its efficacy in practical contexts. The use of triangular fuzzy handling has the potential to significantly enhance existing MCDM approaches such as DEMATEL, AHP, TOPSIS and VIKOR perhaps leading to a revolution in their application and effectiveness. This invention can improve the precision and resilience of decision results, guaranteeing a more knowledgeable and dependable decision-making process. Future study has the potential to explore many applications across a range of disciplines and situations. Furthermore, the integration of the TFGBM operator with other extensions of fuzzy set theory has the potential to enhance decision-making frameworks, allowing for a complete approach to addressing a wider range of real-world situations.

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